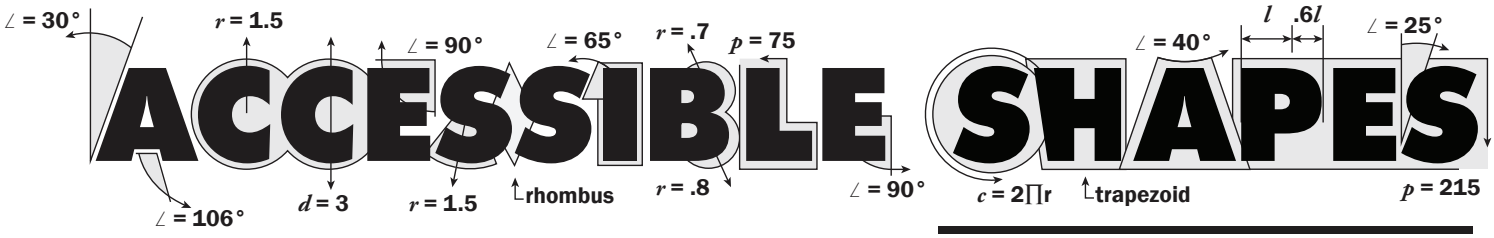

Teacher Guide

WESTERN**RESERVE**
PUBLIC MEDIA

http://www.WesternReservePublicMedia.org/geometry

Contents

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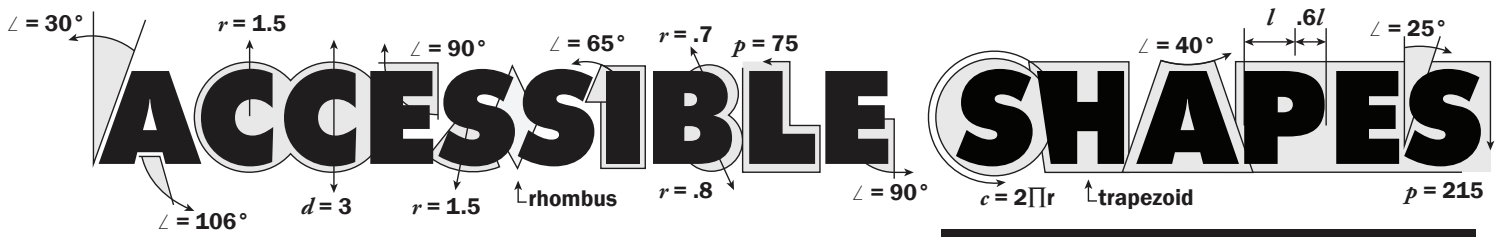


Accessible Shapes: Geometry Overview

http://www.WesternReservePublicMedia.org/geometry

Videos for Accessible Shapes: Geometry

1. **Geometry and Spatial Sense — Professional Development**

   Have you ever lost your car in the parking lot? Or given bad directions? Both tasks require spatial reasoning. Historically, we have considered spatial thinking to be less important than other subjects in the school curriculum. A recent report from the National Research Council says that spatial literacy is important. "In *Learn to Think Spatially*, a study group chaired by Roger M. Downs of Pennsylvania State University states that without explicit attention to [spatial thinking], we cannot meet our responsibility for equipping the next generation of students for life and work in the 21st century." This video explains a specific plan for a hands-on, fun approach to teaching this topic.

2. **Geometry and Spatial Sense — Student**

   This unit asks students to look at building models and determine what they would like from the top, right, left, front and back. Google Earth shows them a building from the top and from the sides so that they can see real-life examples of the activities that they will be doing.

   The video also describes topographic maps, which are maps that show the elevation of a land mass.

3. **Angles, Triangles and Quadrilaterals — Professional Development**

   This video describes the key concept of this unit, which is the understanding of the vocabulary and concepts of angles, triangles and quadrilaterals. Each lesson is built upon the one before it. Formative assessments are placed throughout the lesson so that the teacher can determine if mastery has been achieved.

4. **Angles, Triangles and Quadrilaterals — Student**

   The key concept in this unit is the mastery of the vocabulary and concepts of angles, triangles and quadrilaterals. In the video, students see that these shapes are everywhere – they support the roof over their head and keep bridges from falling down. They gain an understanding of how these shapes impact life.

5. **Surface Area and Volume — Professional Development**

   This video explains the concept of the unit on surface area and volume.

6. **Surface Area and Volume — Student**

   This video provides an introduction to finding the volume and surface area of rectangular prisms and of cylinders. The example of a swimming pool is shown. Is filling the pool the volume or the surface area? Is painting the empty pool the volume or the surface area?

Geometry and Spatial Visualization

Overview

Students will learn about spatial visualization; surface area and volume; and the study of lines, angles and two- and three-dimensional shapes.

Standards Addressed

Geometry: Spatial Visualization

Grade 6

05-07 Benchmark

I. Identify and draw three-dimensional objects from different views (top, side, front and perspective).

Y2003.CMA.S03.G05-07.BI.L06.I07 Visualization and Geometric Models /

07. Build three-dimensional objects with cubes, and sketch the two-dimensional representations of each side; i.e., projection sets.

This unit asks students to look at building models from all sides and determine what model they like best. Students begin with a formative assessment. Because the concepts presented may be a new challenge for students, the lesson plans require them to use building blocks and make structures using a building mat. They then determine the most and least number of blocks needed to create a building. They also try to match a building to one that is given. The final process is a contest to see which group of students can follow a plan to make the building in the least amount of time.

The second part of this activity is to read a topographic map and create a graph when given elevations.

PowerPoint presentations and handouts help the students to better understand these concepts.

Classifying Lines, Angles and Two- and Three-Dimensional Figures

Standards Addressed

Measurement

Grade 6

05-07 Benchmark

D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.

Y2003.CMA.S03.G05-07.BD.L06.I01 Characteristics and Properties /

01. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.

Y2003.CMA.S03.G05-07.BD.L06.I02 Characteristics and Properties /

02. Use standard language to define geometric vocabulary: vertex, face, altitude, diagonal, isosceles, equilateral, acute, obtuse and other vocabulary as appropriate.

Before students can effectively communicate their ideas about geometric shapes, they must learn the proper vocabulary with which to discuss the shapes' properties. This vocabulary is best developed through experience. In the first two lessons, students will draw triangles, review vocabulary as it relates to the shape and then classify triangles. Lessons three through six are designed to review and reinforce concepts of geometric properties and geometric vocabulary of quadrilaterals. The unit project, The Important Book, will take several class periods to complete.

The key concept in this unit is the mastery of the vocabulary of angles, triangles and quadrilaterals. Each lesson is built upon the one before it. Formative assessments are placed throughout the lesson so the teacher can determine if mastery has been achieved.

Surface Area and Volume

Standards Addressed

Measurement

Grade 6

05-07 Benchmark *G. Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes.*

Y2003.CMA.S02.G05-07.BG.L06.I01 Measurement Units /

01. Understand and describe the difference between surface area and volume.

05-07 Benchmark *F. Analyze and explain what happens to area and perimeter or surface area and volume when the dimensions of an object are changed.*

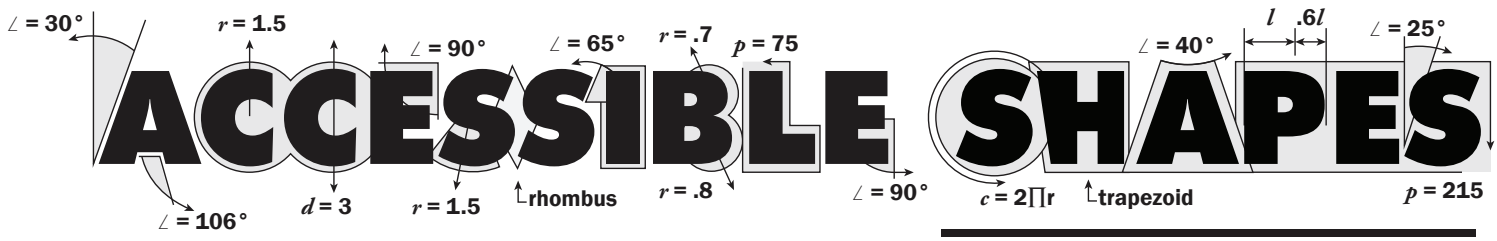
Y2003.CMA.S02.G05-07.BF.L06.I01 Measurement Units /

01. Understand and describe the difference between surface area and volume.

This unit helps students to understand the difference between volume and surface area. They begin with formative assessments for general vocabulary and general assessment. They then learn how to find the area and perimeter of rectangular prisms, and the area and circumference of cylinders. They also are introduced to volume and surface area of rectangular prisms and cylinders through the use of PowerPoint presentations and student handouts.

As a final project, the students use a sheet of 8.5" x 11" paper to make a rectangular prism in two different ways. They must then determine which has a greater volume and which has a greater surface area. The next part of the lesson asks the students to repeat the process, but this time to make a cylinder two different ways to determine which has a greater volume. Popcorn is the tool that will help them visually see the results.

The project is followed by a summative assessment.



Geometry and Spatial Sense

http://www.WesternReservePublicMedia.org/geometry

Geometry and Spatial Sense Overview

Have you ever lost your car in the parking lot? Or given bad directions? Both tasks require spatial reasoning. Historically we have considered spatial thinking to be less important than other subjects in the school curriculum. A recent report from the National Research Council argues the importance of spatial literacy.

> "In 'Learn to Think Spatially,' a study group chaired by Roger M. Downs of Pennsylvania State University states that without explicit attention to [spatial thinking], we cannot meet our responsibility for equipping the next generation of students for life and work in the 21st century."

Newcombe, Nora S. "A Plea for Spatial Literacy." *Chronicle of High Education* 52.26 (2006): Print.

Spatial thinking gives us unique insights into various science, technology, engineering and math disciplines such as visualizing the processes involved in the earth's formation, visualizing images from MRI tests, and reading graphs, charts and diagrams.

There is strong evidence that women perform more poorly than men on visualization tasks. There is also some evidence that differences exist based on socioeconomic status. Increasing evidence shows that we can teach spatial-thinking skills. Newcombe did a meta-analysis of the studies in the last 15 years and found that teachers can improve spatial sense through practice with specific tasks and some computer games.

Teaching spatial sense reduces the differences related to gender and socioeconomic levels and helps people to participate fully in our technological society.

This unit deals with the following standard:

Geometry and Spatial Sense

Grade 6

05-07 Benchmark *I. Identify and draw three-dimensional objects from different views (top, side, front and perspective).*

Y2003.CMA.S03.G05-07.BI.L06.107 Visualization and Geometric Models /

07. Build three-dimensional objects with cubes, and sketch the two-dimensional representations of each side: i.e. projection sets.

In this module, students will begin by taking a formative assessment to assess their current learning on this topic. The teacher will watch a video explaining the purpose for studying this topic and the lessons in the module. The students will watch a video about this topic, do the lessons in the teacher guide, create a final project and finish with a summative assessment about spatial sense.

Introduction to Spatial Sense

Overview

In this lesson students will become familiar with the tools used in spatial visualization, create a building and draw it from all sides using a building mat, draw all sides and draw the conclusion that the right side is a mirror image of the left side and the front is a mirror image of the back side.

Standard Addressed

Geometry and Spatial Sense

Grade 6

05-07 Benchmark

I. Identify and draw three-dimensional objects from different views (top, side, front and perspective).

Y2003.CMA.S03.G05-07.BI.L06.107 Visualization and Geometric Models /

07. Build three-dimensional objects with cubes, and sketch the two-dimensional representations of each side: i.e. projection sets.

Materials

- Six blocks per group of students
- One building mat per group
- *Building a Climbing Wall* student handout
- *Mirror Image* student handout

Procedure

Part 1

1. Divide the class into groups of two or three.

2. Distribute the *Building a Climbing Wall* handout and read the scenario:

   "You have volunteered to be on a committee to build a climbing wall for a park. You have been given a picture of the base of the building. The number on the block tells how many blocks high the climbing wall will be. You want to visualize how the wall will look, so you volunteer to draw the way the wall will look from all sides."

3. Tell the students to select one person from their group to come to the front and get a building mat and bag of blocks.

4. Explain that the view of the building is the top view, as if you were flying over it in a balloon. The number represents the height of each part of the wall.

5. Have students make the wall as shown on the scenario page. Walk around and check that each group has successfully done this.

6. Now ask each group to draw what this would look like if they were looking at it from the front, back, left and right.

   Answers:

Front	Back	Right	Left
			

7. Have students draw their plans on the board and make sure that each group has the correct solutions.

8. Ask the students to see if they can draw a conclusion from their building plans. From now on, the students will only have a building mat with base, front and right sides shown. ***Sample conclusion:*** *The front is a mirror image of the back and the right is a mirror image of the left. Therefore, if they know the base, the front and the right, they can accurately create the building.*

9. Using the PowerPoint presentation *Mirror Image,* have the students number their paper to four and draw the mirror image of the slides shown. They can check their work as they progress. An alternative plan is to use the *Mirror Image* student handout.

Part 2

Materials

- 20 blocks per group of students
- One building mat per group
- *Buildings* student handout
- *Matching Buildings* student handout

1. Instruct the students to construct buildings using the blocks and the design images. Distribute the *Buildings* student handout. There are several possible solutions for the second and third buildings.

2. The next activity is to determine which building matches the building in the box. Distribute the *Matching Buildings* student handout. Allow the students to work with a partner if they'd like to. This activity could be used as an assessment to determine student understanding of the building concept. You could either allow or not allow the use of blocks. *Answer: The correct answer is A. In B, the right side is incorrect and in C, the front is incorrect.*

Part 3

Materials

- Six blocks per group of students
- One building mat per group
- *Most and Least* student handout

1. Have the students use the same building plan to determine if the buildings will always be exactly alike. Using the *Most and Least* handout, students can either work alone or with a partner to build the building and then record the greatest and least number of blocks they could use and retain the same plan.

Answers:

1. *Most 11; least 9.*

2. *Most 20; least 16.*

3. *Answers will vary, but most will say that they actually added and subtracted blocks to find the correct answer.*

4. *There could be different buildings with the least amount of blocks because they could be placed in different areas. There could only be one building with the most blocks.*

Part 4

Materials

- 75 blocks per group of students
- One building mat per group
- *Building Contest* student handout

1. This activity could be used as an authentic assessment or as a contest. Students can be placed into groups of 4 or 5. This is because a lot of blocks are necessary to do this activity.

   Answer: The building with the least amount of blocks uses 60 blocks and the one with the most uses 72. The pictures below show possible buildings that could be made, but they do not show all possible answers.

   Using the building plan, instruct the students to make the building. The activity can be timed. There are several possibilities, so check each group's building to see if it matches the plan. The prize for winning the contest can be simply to write the winners' names on the board or can be more elaborate as you see fit.

Evaluation

If ***Matching Buildings*** is used as an evaluation, 10 points can be given for the correct answer, plus 10 points each for selecting why B and C were incorrect.

The building project can be used as an authentic assessment or the summative assessment can be used as the evaluation tool.

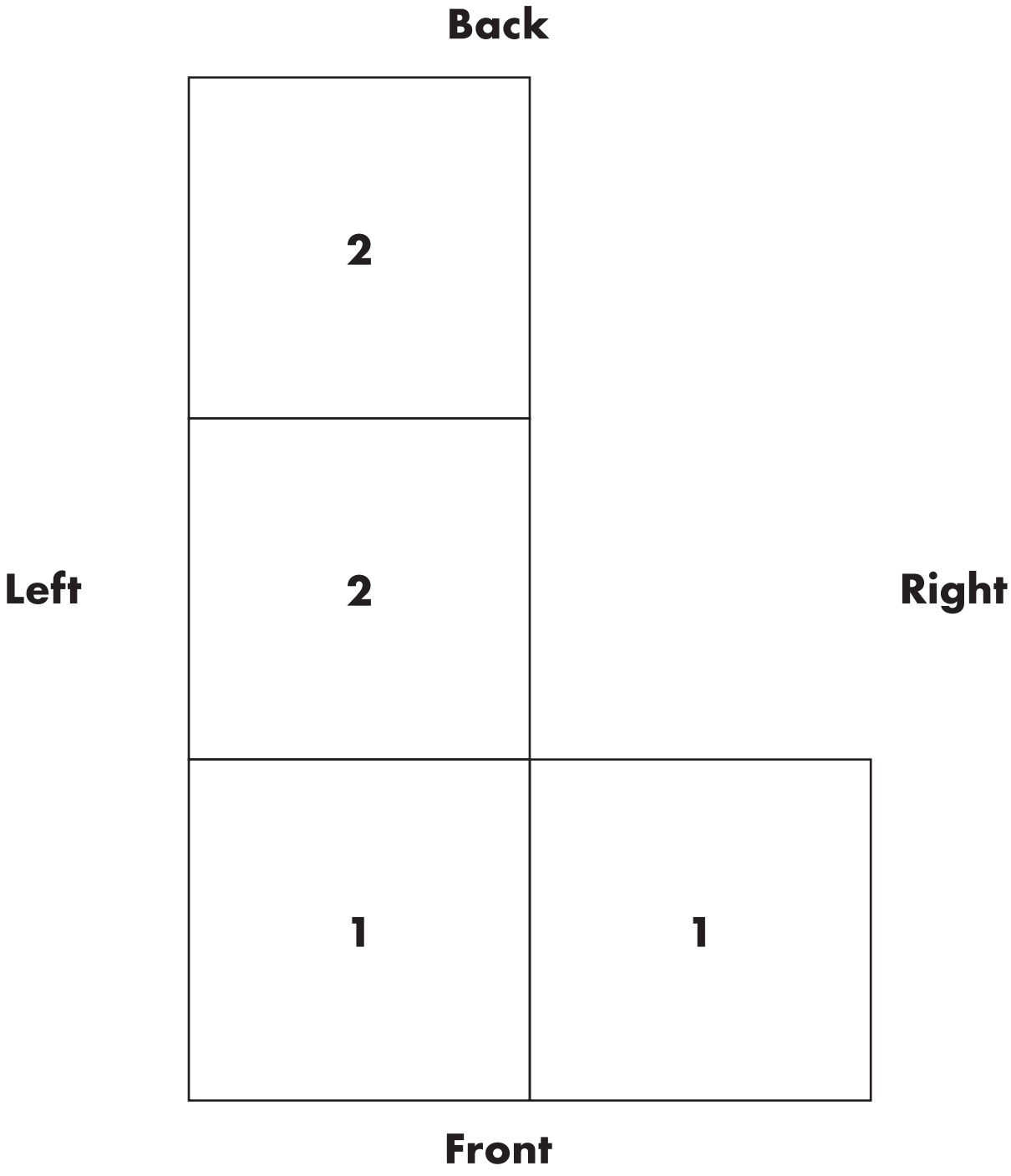
Resources

There are two sources that give complete instruction in using spatial visualization with students:

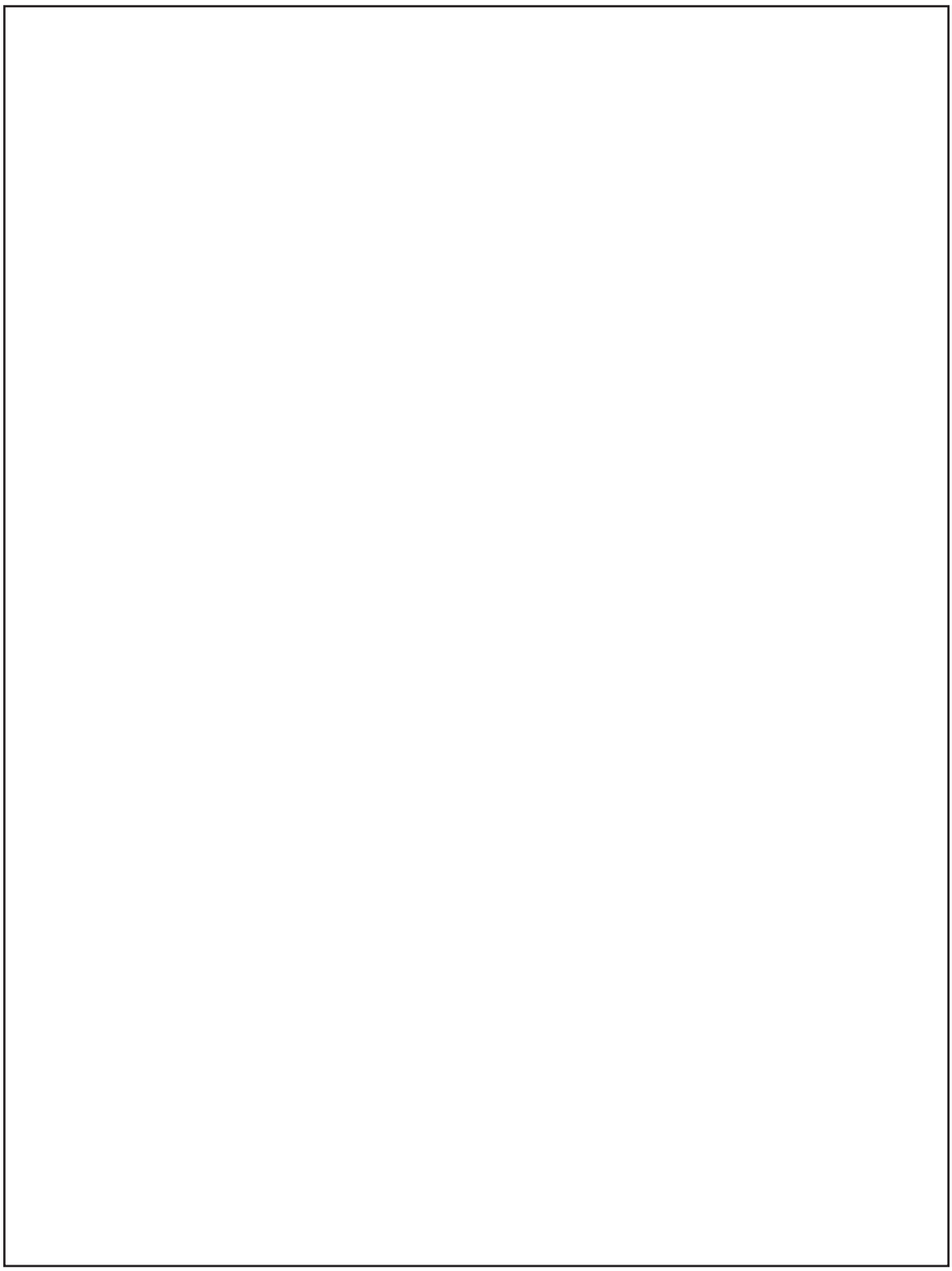
- Winter, Mary Jean, et al. *Spatial Visualization: Middle Grades Mathematics Project*. Menlow Park, CA: Addison-Wesley Publiching Company, 1986. Print.
- "Isometric Drawings." *Illuminations: Resources for Teaching Math*. NCTM, n.d. Web. 10 Sep. 2010. <http://illuminations.nctm.org.>

Building a Climbing Wall

You have volunteered to be on a committee to build a climbing wall for a park. You have been given a picture of the base of the building. The number on the block tells how many blocks high the climbing wall will be. You want to visualize how the wall will look, so you volunteer to draw the way the wall will look from all sides.

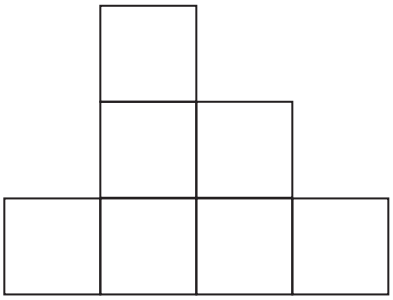
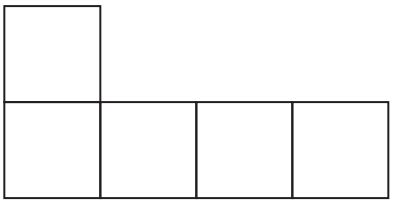
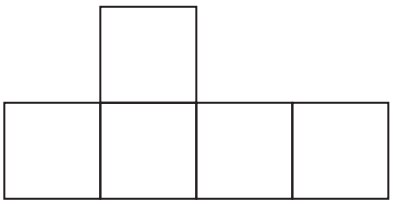
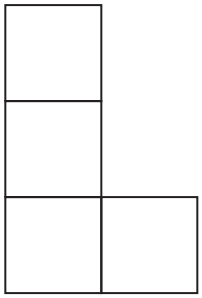


Building Mat



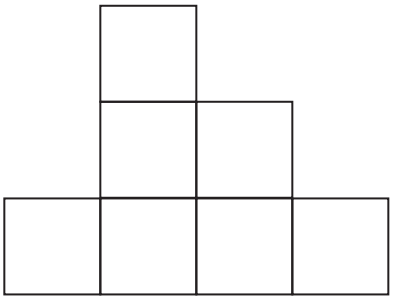
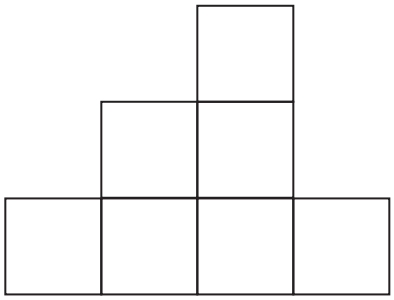
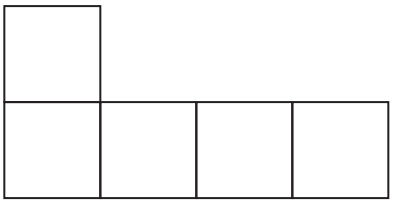
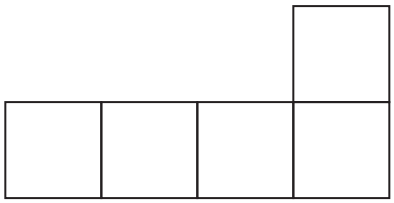
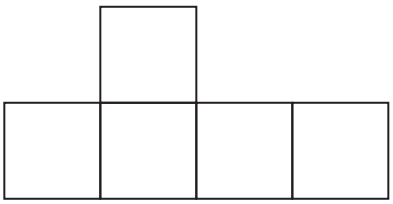
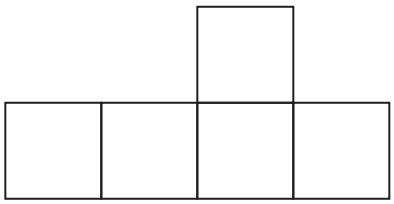
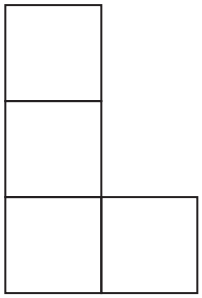
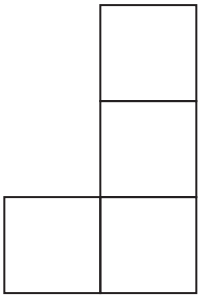
Mirror Image

Draw the mirror image of the figure on the left.

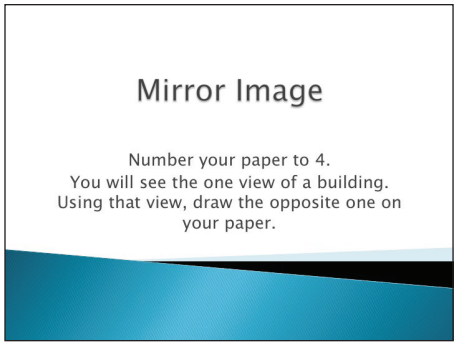
1.  Front	Back
2.  Right	Left
3.  Left	Right
4.  Back	Front

Mirror Image Answers

Draw the mirror image of the figure on the left.

1.  Front	 Back
2.  Right	 Left
3.  Left	 Right
4.  Back	 Front

Mirror Images PowerPoint Presentation



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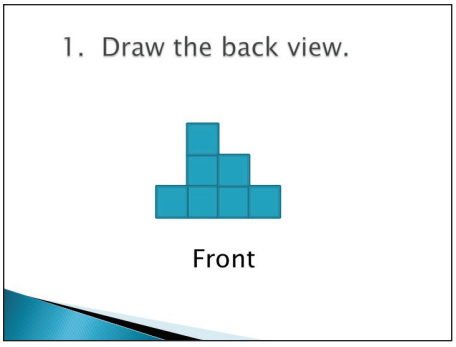

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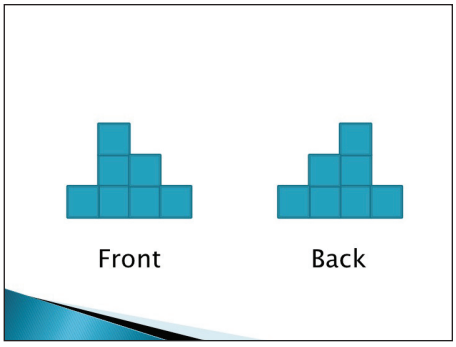

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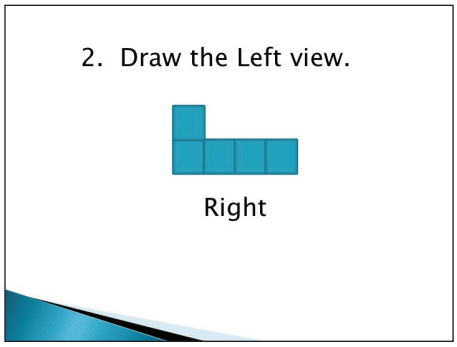

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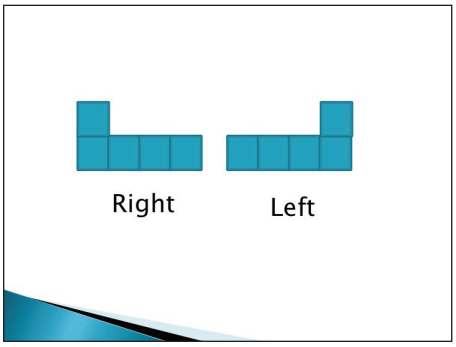

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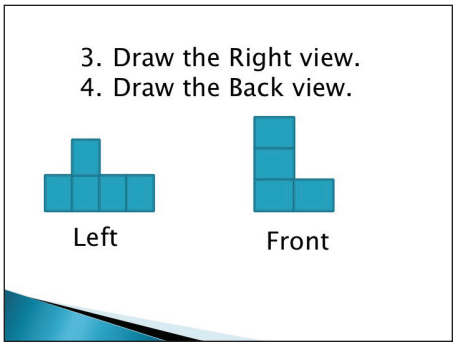

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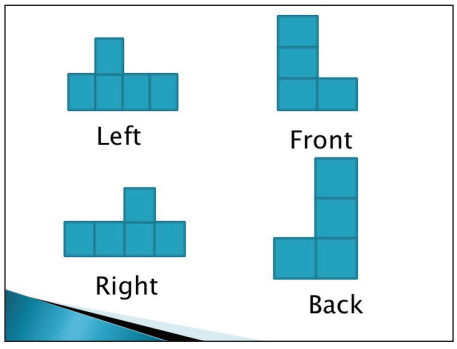
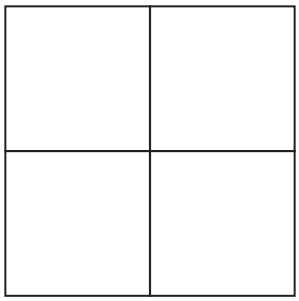
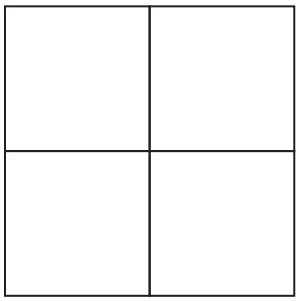
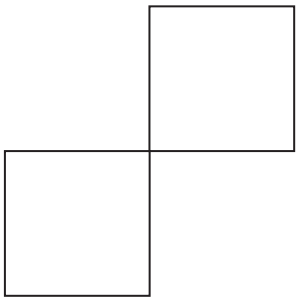
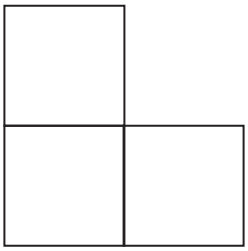
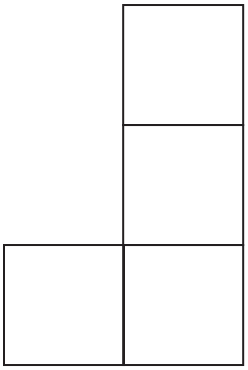
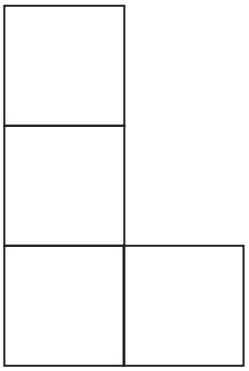
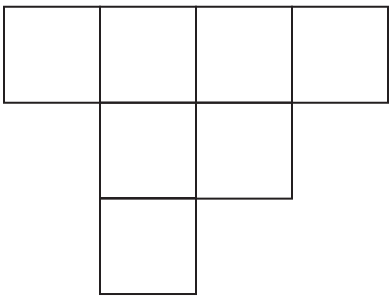
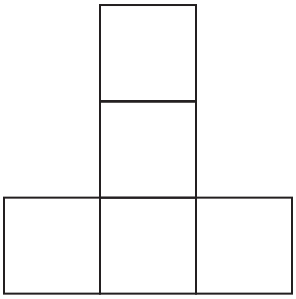
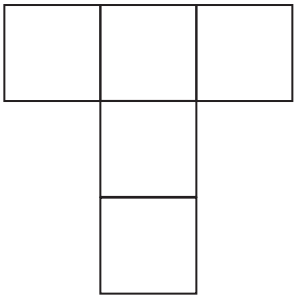


Slide 7

Buildings

Make these three building designs using blocks. Have the teacher initial your sheet when you have completed each one.

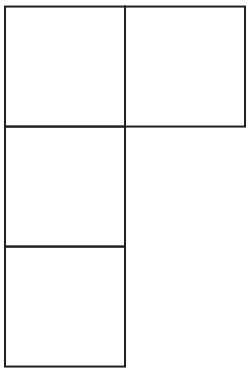
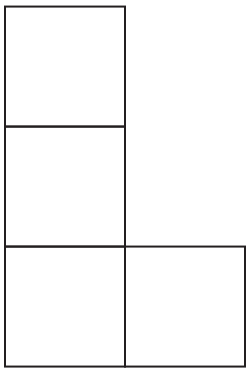
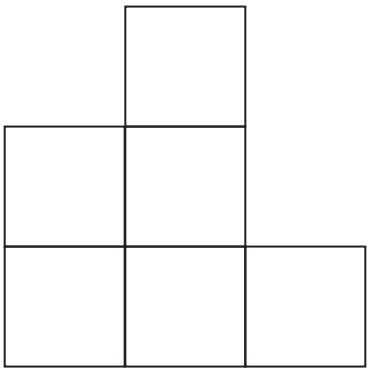
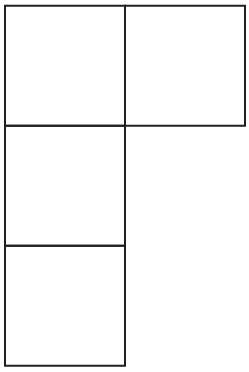
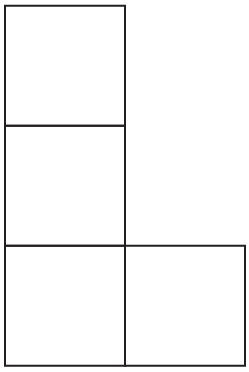
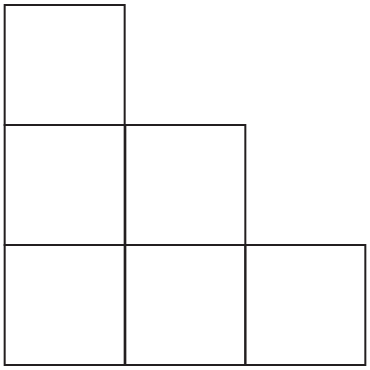
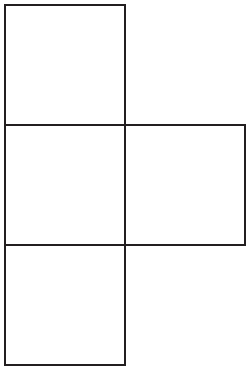
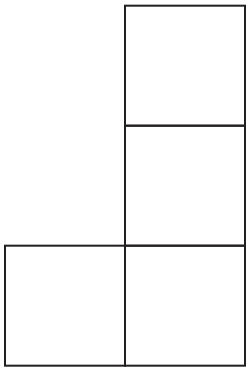
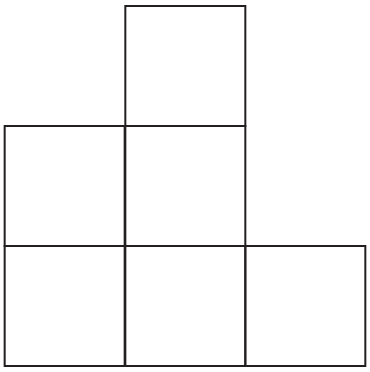
Base	Front	Right
1. 		 Initials _____
2. 		 Initials _____
3. 		 Initials _____

1	1
3	
2	

Name _____

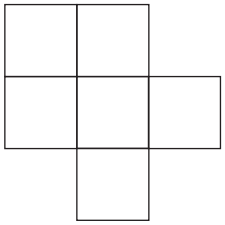
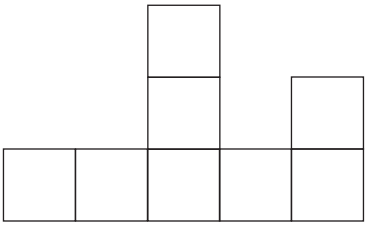
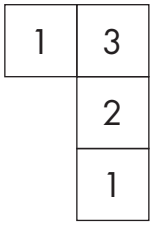
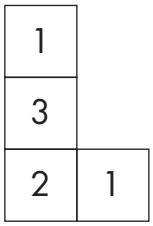
Matching Buildings

Select the plan that matches the building on the right. Write a brief paragraph as to why you selected that plan.

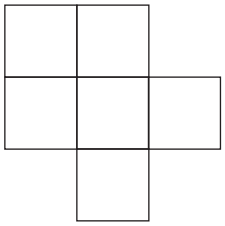
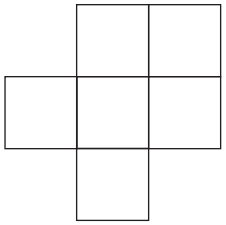
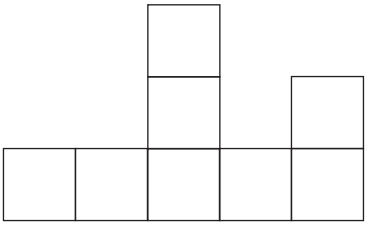
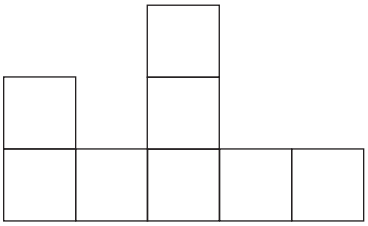
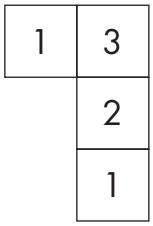
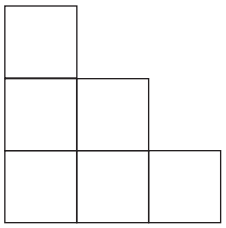
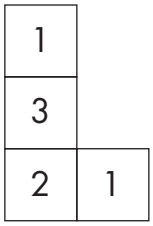
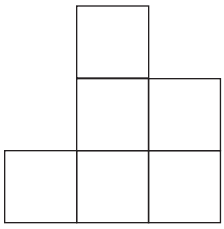
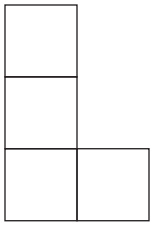
Base	Front	Right
A. 		
B. 		
C. 		

Name _____

Formative Assessment

This is the right view. Show the left view. 	
This is the back view. Show the front view 	
This is the base. Show the left view. 	
This is the base. Show the left and the front. 	Left Front
Tell what a topographic map is and why you would use it.	

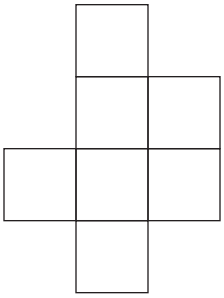
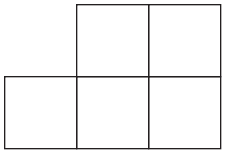
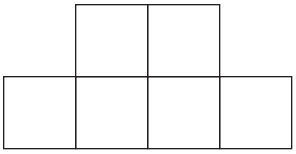
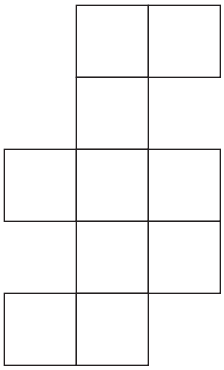
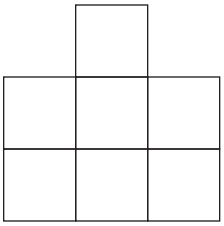
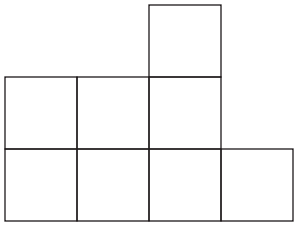
Formative Assessment

This is the right view. Show the left view. 	
This is the back view. Show the front view 	
This is the base. Show the left view. 	
This is the base. Show the left and the front. 	Left  Front 
Tell what a topographic map is and why you would use it.	• Shows the difference in elevation between any two points • Shows the steepness of the land • Would be good to help hikers • Shows rivers, mountains, valleys, hills, etc.

Name _____

Most and Least

You will be given a base plan, a front and a right. What is the most number of blocks that could be used to make this building? What is the least number of blocks?

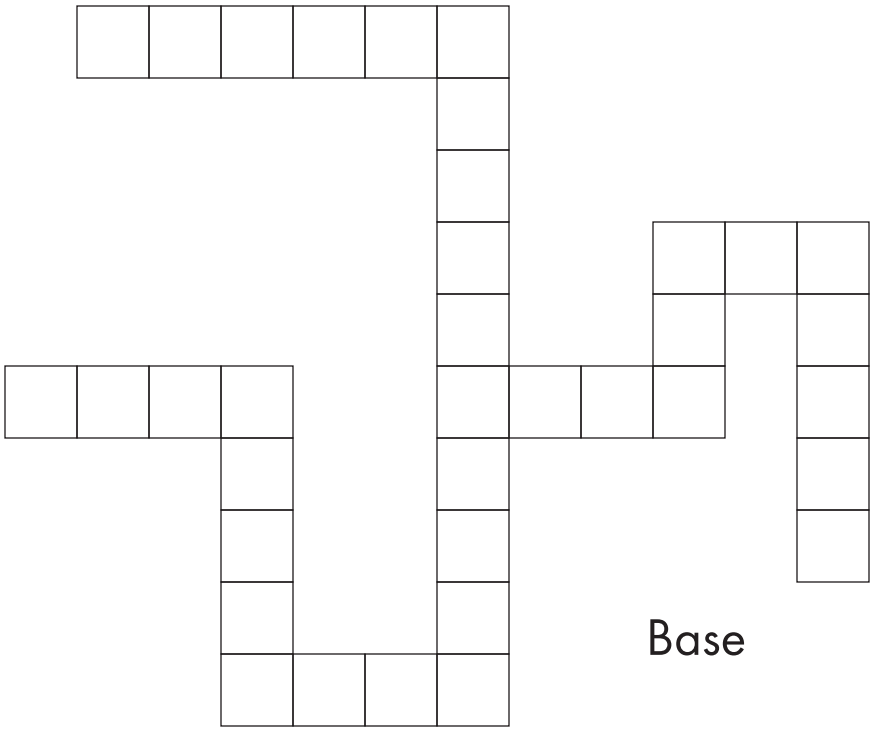
Base	Front	Right	No. of Blocks
1. 			Most _____ Least _____
2. 			Most _____ Least _____

3. How did you figure out how many to add?

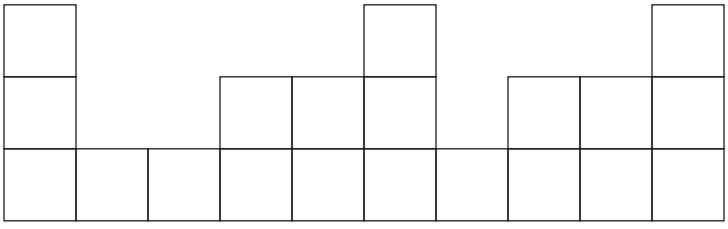
4. Could you have more than one building with the most blocks? The least blocks? Why?

Building Contest

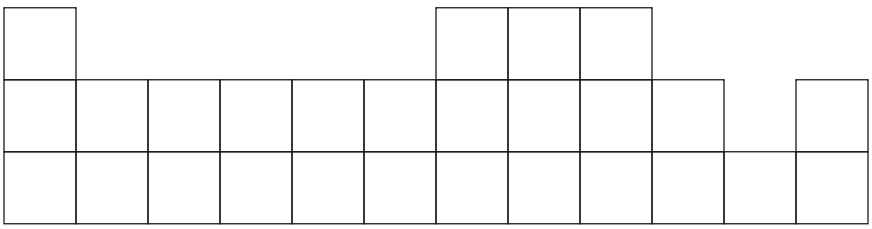
Team Members _____



Base



Front



Right

Completed the building correctly:

_____

Teacher initial above!

Number of blocks in the base

_____

Least number of blocks

_____

Most number of blocks

_____

Time to complete building

_____

Topographic Maps

A topographic map is a large-scale map showing relief and man-made features of a portion of a land surface. It shows the position, relation, size, shape and elevation of these features. It is a **representation, on a flat surface, of a part of the Earth's surface drawn to scale.**

Vocabulary

Source: http://www.ucmp.berkeley.edu/fosrec/Metzger1.html

Topographic map: The difference in elevation between any two points. Features can include: 1. hills, valleys, mountains, etc.; 2. water features including lakes, ponds, etc.; and 3. cultural features that are man-made such as bridges, buildings, etc.

Contour lines: An imaginary line on the topographic map that connects points of the same elevation on the earth's surface.

Contour intervals: Shows the difference in elevation between adjacent contour lines. It can be large or small, but it is always consistent on a map.

Index contour: Usually every fifth contour line is printed heavier than the others and tells the elevation above sea level.

Benchmarks: Points marked by brass plates that are fixed permanently on the ground. The topographic map will show the elevation preceded by the letters BM.

Scale: Expresses the relationship between distance on the map and the true distance on the earth's surface. This is generally a fraction or a ratio such as 1/24,000. The numerator represents the map distance. The denominator represents the actual distance on the ground.

Basic Information About Contour Lines

1. Contour lines are widely spaced on gentle slopes.
2. Contour lines are closely spaced on steep slopes.
3. Contour lines do not cross or intersect each other.
4. Contour lines eventually close. They may close on the map or beyond the margin of the map.
5. The top of a hill is higher than the closest contour line.
6. The bottom of a depression is lower than the closest contour line.
7. Sometimes colors are used to represent different items on the map. Blue = water features; green = woodlands; red = urban areas; black = man-made works; brown = contour lines; purple = revised versions.

Using Topographic Maps

Overview

In this lesson the students will learn about topographic maps by watching a PowerPoint presentation. They will then use what they have learned to answer questions about a topographic map and try to show how this two-dimensional map can show elevation.

Standard Addressed

Geometry and Spatial Sense

Grade 6

05-07 Benchmark *I. Identify and draw three-dimensional objects from different views (top, side, front and perspective).*

Y2003.CMA.S03.G05-07.BI.L06.I07 Visualization and Geometric Models /

07. Build three-dimensional objects with cubes, and sketch the two-dimensional representations of each side; i.e., projection sets.

Materials

- *Topographic Maps* PowerPoint presentation
- *Reading Topographic Maps* student handout
- Graph paper

Procedure

1. Ask the students if they had ever been hiking. If they have, ask them how they knew where they were going. Did they use a compass, a map, etc.? Ask if they have ever seen a topographic map. Tell them that these maps are also called topo maps.

2. Show a topographic map to the class or print out a copy for each student.

3. Use the *Topographic Maps* PowerPoint presentation with the students.

4. After the maps have been discussed, break the students into groups of two or three. Give each person a copy of the *Reading Topographic Maps* student handout. Have them work together to answer the questions and make a graph showing the elevation view of the map. This is most easily done if they cut the map out of the sheet, fold it in half from north to south and then mark the distances on the x-axis. They can then transfer the elevations to the correct spot on the graph.

Answers for student handout

1. 100 feet.

2. The north side is steepest because the contour lines are closest together.

3. The biggest space with the least increase in elevation is on the southwest corner.

4. Topographic Map

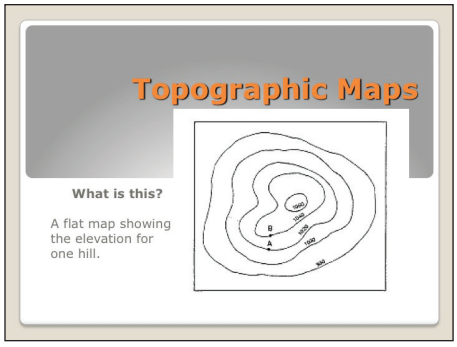
5. Go over the answers to the questions and then have the students share their graphs of how the land would look at that point.

Evaluation

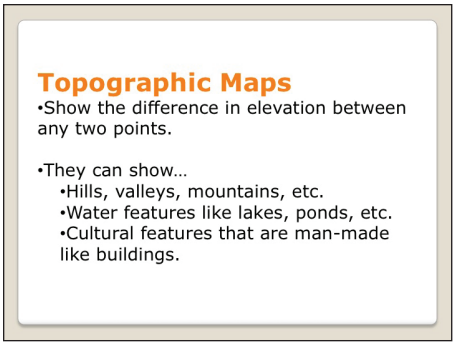
This lesson is an introduction to topographic maps. It pairs nicely with the concept of looking at a flat drawing of buildings from the base, the front and the right. It allows students to see how a flat drawing can illustrate elevation.

Student handouts can be collected and graded if a grade or percent is needed.

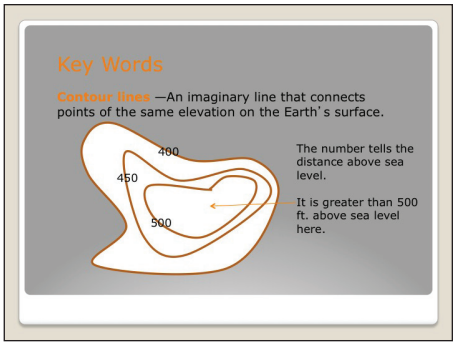
Topographic Maps PowerPoint Presentation



Slide 1



Slide 2



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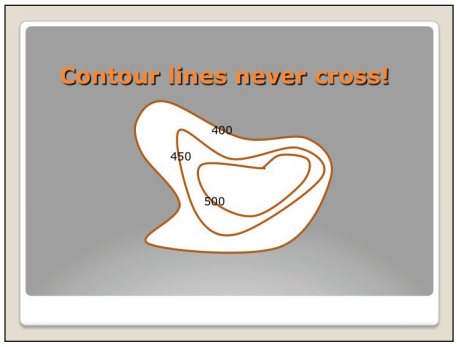

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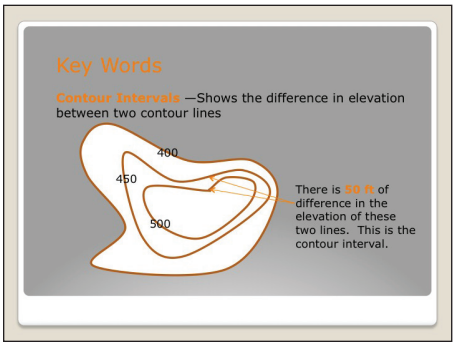
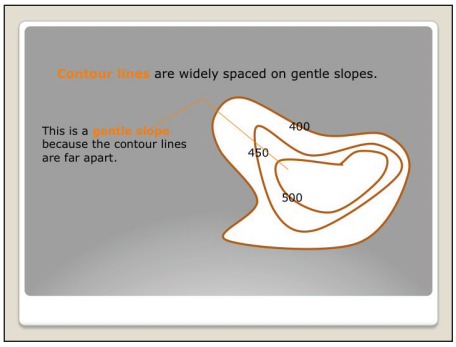
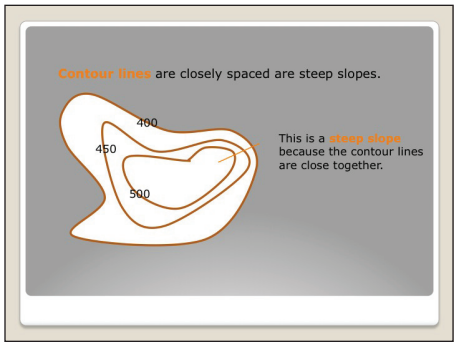


Slide 5



Slide 6



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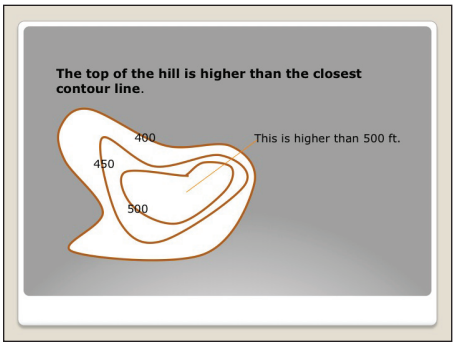
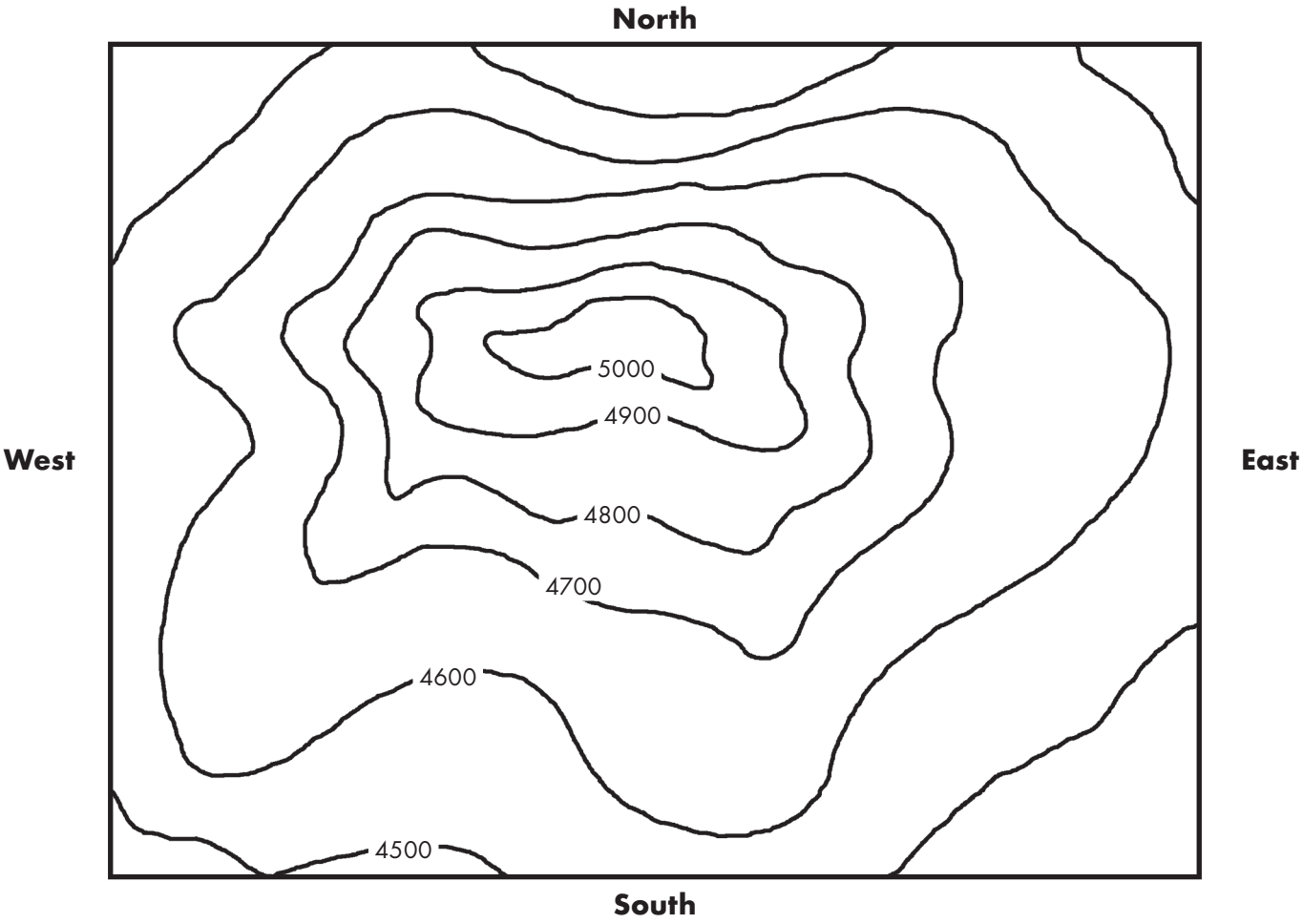


Slide 8

Reading Topographic Maps

Answer the questions below using information you get from the topographic map.

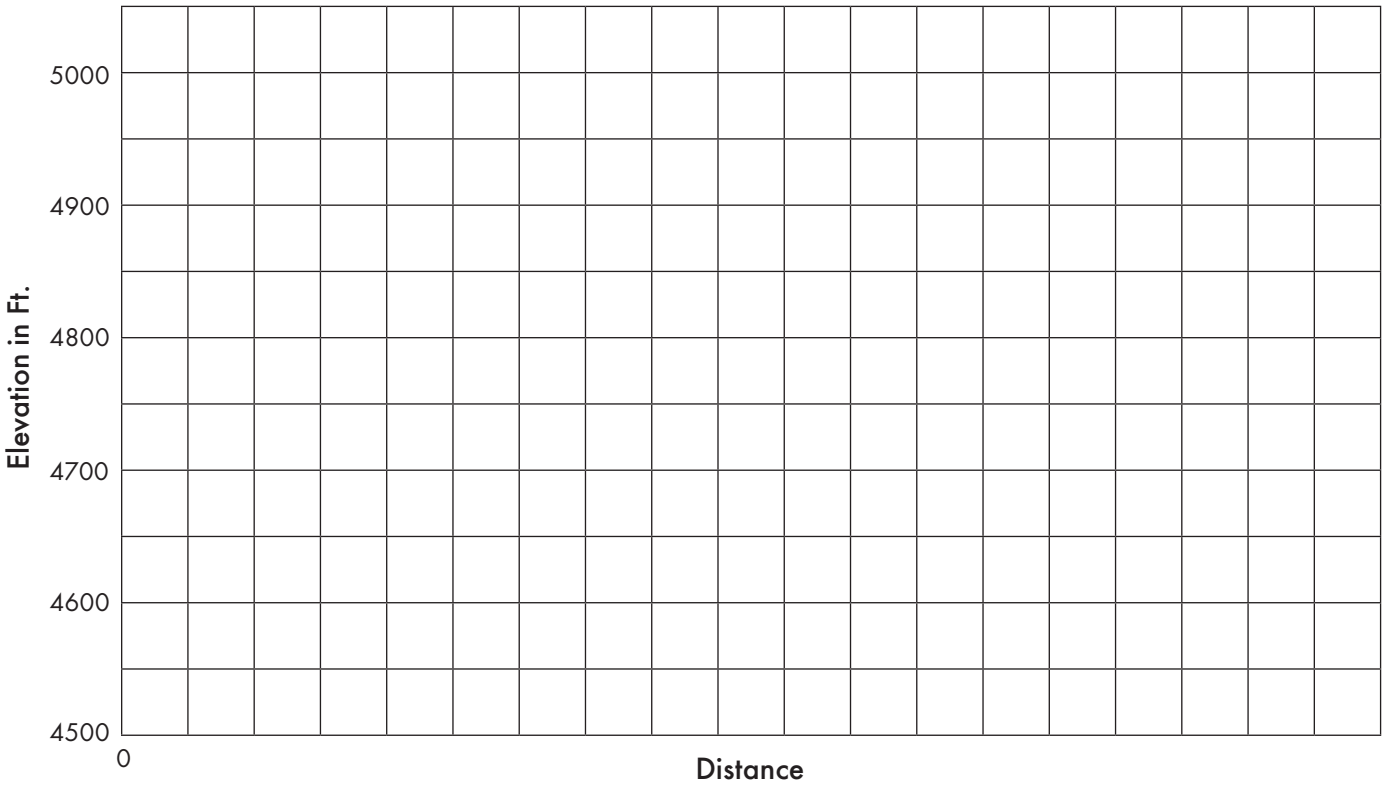


1. How many feet are between each contour line?

2. Which side has the steepest slope? Why do you think that side is the steepest?

3. You want to build a camp for children on this land. Where would you put it and why would you put it there?

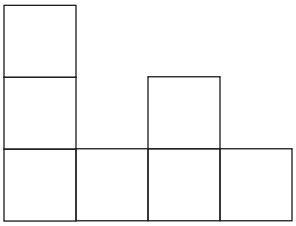
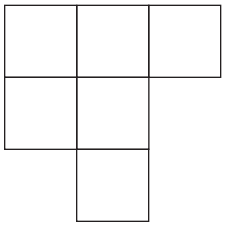
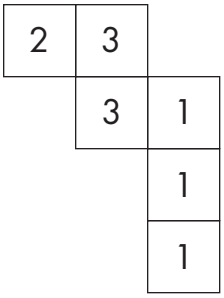
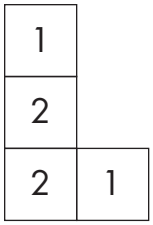
4. If you were to fold this map in half from north to south, what would the side view of the map look like?



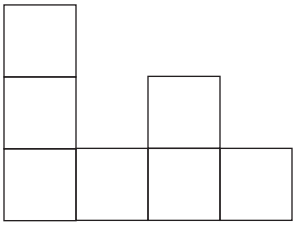
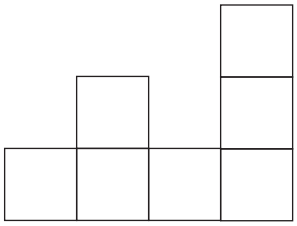
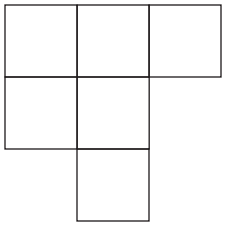
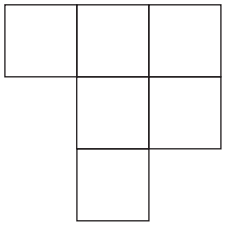
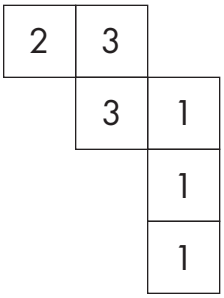
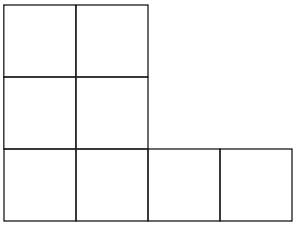
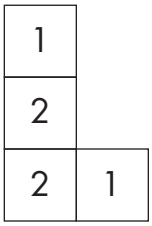
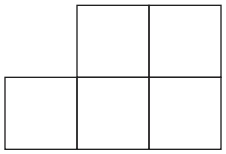
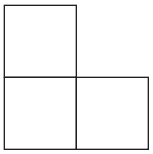
Graph for topographic map elevations (side view). No distance measure is given on the original map.

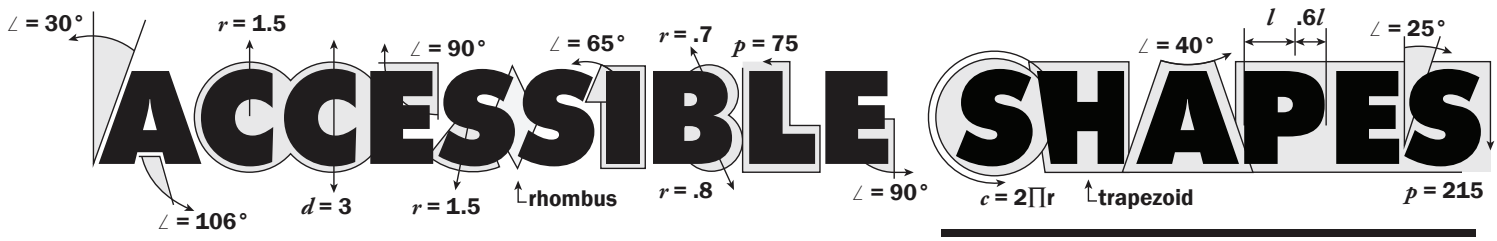
Name _____

Summative Assessment

This is the right view. Show the left view. 	
This is the back view. Show the front view 	
This is the base. Show the left view. 	
This is the base. Show the left and the front. 	Left Front
Tell what a topographic map is and why you would use it.	

Summative Assessment

This is the right view. Show the left view. 	
This is the back view. Show the front view 	
This is the base. Show the left view. 	
This is the base. Show the left and the front. 	Left  Front 
Tell what a topographic map is and why you would use it.	• Shows the difference in elevation between any two points • Shows the steepness of the land • Would be good to help hikers • Shows rivers, mountains, valleys, hills, etc.



GEOMETRY

Angles, Triangles and Quadrilaterals

http://www.WesternReservePublicMedia.org/geometry

Angles, Triangles and Quadrilaterals Overview

Before students can effectively communicate their ideas about geometric shapes they must have a vocabulary with which to discuss the properties of those shapes. This vocabulary is best developed through experience. In lessons 1 and 2 students will draw triangles, review vocabulary as relates to triangles and then classify triangles. Lessons 3, 4, 5, and 6 are designed to review and reinforce concepts of geometric properties and geometric vocabulary of quadrilaterals. The unit project will be introduced in lesson 7 and that project will take several class periods.

Lesson 1: What Makes a Triangle a Triangle? Drawing and Sorting Triangles

Students will use a sorting activity to review and reinforce definitions of geometric vocabulary.

Lesson 2: Classifying Triangles by Side Length and Angle

Students will use a sorting activity to explore and analyze these types of triangles: scalene, isosceles, equilateral, acute, right and obtuse.

Lesson 3: Building Quadrilaterals

Students will use manipulatives to make quadrilaterals. Students will analyze the shapes that they have made and use appropriate vocabulary to describe and classify them.

Lesson 4: Sorting Quadrilaterals

Students will analyze the properties of quadrilaterals and will use appropriate vocabulary to describe and classify quadrilaterals into groups and subgroups.

Lesson 5: Organizing Quadrilaterals, A Quadrilateral Family Tree

Students will analyze the properties of quadrilaterals and use their understanding of those properties to organize quadrilaterals into a family tree.

Lesson 6: Analyzing the Quadrilateral Family Tree

Students will analyze the properties of quadrilaterals and use their understanding of those properties to classify the shapes.

Lesson 7: The Important Book

Using their understanding of the properties of triangles and quadrilaterals, the students will create a book.

Benchmarks and Standards

Measurement

Grade 6

05-07 Benchmark

D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.

Y2003.CMA.S03.G05-07.BD.L06.I01 Characteristics and Properties /

01. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.

Y2003.CMA.S03.G05-07.BD.L06.I02 Characteristics and Properties /

02. Use standard language to define geometric vocabulary: vertex, face, altitude, diagonal, isosceles, equilateral, acute, obtuse and other vocabulary as appropriate.

Y2003.CMA.S02.G0507.BC.L06.I03 Use Measurement Techniques and Tools /

03. Estimate perimeter or circumference and area for circles, triangles and quadrilaterals, and surface area and volume for prisms and cylinders by:
    - Estimating lengths using string or links, areas using tiles or grid and volumes using cubes;
    - Measuring attributes (diameter, side lengths or heights) and using established formulas for circles, triangles, rectangles, parallelograms and rectangular prisms.

Lesson 1: What Makes a Triangle a Triangle?

Drawing and Sorting Triangles

Overview

Students will use a sorting activity to review and reinforce definitions of geometric vocabulary.

Benchmarks and Standards

Measurement

Grade 6

05-07 Benchmark

D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.

Y2003.CMA.S03.G05-07.BD.L06.I01 Characteristics and Properties /

01. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.

Y2003.CMA.S03.G05-07.BD.L06.I02 Characteristics and Properties /

02. Use standard language to define geometric vocabulary: vertex, face, altitude, diagonal, isosceles, equilateral, acute, obtuse, and other vocabulary as appropriate.

Materials

- One sheet of centimeter graph paper per student
- Centimeter rulers
- Scissors

Procedure

1. Pass out one sheet of centimeter graph paper to each student.
2. Instruct the students to divide each piece of paper into sixths by folding it in half lengthwise and then into thirds.

3. Review properties of triangles. Tell students to draw one triangle in each sixth of the paper, filling each space. Each triangle should look different.

4. Instruct the students cut out the triangles.

5. Have the students work in pairs and use their triangles to identify the following: side, vertex, vertices, angle, polygon and triangle. Use class discussion to arrive at simple and "child-friendly" definitions and write these definitions on chart paper. Include a drawing with each definition. See glossary at end of lesson.

6. Have the pairs of students combine into groups of four.

7. Ask students to sort all the triangles. They should have criteria for the categories of triangles that they create.

8. In turn, each group should share their work with the class. Discuss geometric vocabulary as students use it to describe their shapes, including congruent (sides and angles), right angle, perpendicular, acute angle and obtuse angle. Add definitions to the glossary on chart paper and include a drawing.

9. Students should save their shapes for the next lesson.

Glossary

Acute angle: an angle that measures less than 90 degrees

Angle: two rays that share an endpoint

Congruent: having exactly the same size and shape

Obtuse angle: an angle that measures greater than 90 degrees

Perpendicular: forming right angles

Polygon: a closed figure formed from line segments that meet only at their endpoints

Right angle: an angle that measures 90 degrees

Side: a line segment connected to other segments to form a polygon

Triangle: a three-sided polygon

Vertex: the point at which two line segments, lines, or rays meet to form an angle

Vertices: plural of vertex

Reference: Math on Call, A Mathematics Handbook

Lesson 2: Classifying Triangles By Side Length and Angle

Overview

Students will use a sorting activity to explore and analyze types of triangles including scalene, isosceles, equilateral, acute, right and obtuse.

Benchmarks and Standards

Measurement

Grade 6

05-07 Benchmark

D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.

Y2003.CMA.S03.G05-07.BD.L06.I01 Characteristics and Properties /

01. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.

Y2003.CMA.S03.G05-07.BD.L06.I02 Characteristics and Properties /

02. Use standard language to define geometric vocabulary: vertex, face, altitude, diagonal, isosceles, equilateral, acute, obtuse and other vocabulary as appropriate.

Y2003.CMA.S02.G0507.BC.L06.I03 Use Measurement Techniques and Tools /

03. Estimate perimeter or circumference and area for circles, triangles and quadrilaterals, and surface area and volume for prisms and cylinders by:
    - Estimating lengths using string or links, areas using tiles or grid, and volumes using cubes;
    - Measuring attributes (diameter, side lengths or heights) and using established formulas for circles, triangles, rectangles, parallelograms and rectangular prisms.

Materials

- Triangles from previous lesson
- Chart paper with glossary from previous lesson
- *Sorting Triangles by Side Length and Angle Measurement* chart
- Masking tape

Procedure

1. Review glossary from previous lesson.
2. Organize students into groups.
3. Draw the *Sorting Triangles by Side Length and Angle Measurement* chart on the board.
4. Ask students to analyze the triangles that they drew in the previous lesson. Do any of their triangles fit in the chart? Where do they fit? Can a triangle fit in more than one place?
5. Invite students to tape one of their triangles to the chart on the board and give a rationale for the placement. The goal during this activity is to arrive at an understanding of the properties of each of the types of triangles on the chart: acute, right, obtuse, equilateral, isosceles and scalene. Students should also be able to recognize that triangles can be sorted by angle measurement and side length. Time permitting, students can put the rest of their triangles on the chart.
6. Invite the students to analyze the chart. Are there any empty spaces? Why? Can triangles be drawn to fit those empty spaces?
7. Choosing triangles from the chart, discuss the types of triangles and their properties. Arrive at a definition of each type of triangle. Add these definitions to the glossary and draw an example with each definition.
8. Pass out copies of blank charts. Give students time to draw correct triangles in the spaces.

Sorting Triangles By Side Length and Angle Measurement

	Equilateral	Isosceles	Scalene
Acute			
Right			
Obtuse			

Glossary

Acute angle: an angle that measures less than 90 degrees

Acute triangle: a triangle with no angle measuring 90 degrees or more

Angle: two rays that share an endpoint

Congruent: having exactly the same size and shape

Equilateral triangle: a triangle with three congruent sides

Isosceles triangle: a triangle with two congruent sides

Obtuse angle: an angle that measures greater than 90 degrees

Obtuse triangle: a triangle whose largest angle measures greater than 90 degrees

Perpendicular: forming right angles

Polygon: a closed figure formed from line segments that meet only at their endpoints

Right angle: an angle that measures 90 degrees

Right triangle: a triangle with a right angle

Scalene triangle: a triangle with no congruent sides

Side: a line segment connected to other segments to form a polygon

Triangle: a three-sided polygon

Vertex: the point at which two line segments, lines, or rays meet to form an angle

Vertices: plural of vertex

Reference: Math on Call, A Mathematics Handbook

Lesson 3: Building Quadrilaterals

Overview

Students will use manipulatives to make quadrilaterals. Students will analyze the shapes that they have made and use appropriate vocabulary to describe and classify them.

Benchmarks and Standards

Measurement

Grade 6

05-07 Benchmark

D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.

Y2003.CMA.S03.G05-07.BD.L06.I01 Characteristics and Properties /

01. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.

Materials

- *Guess My Rule!* handout prepared for overhead projection
- Glossary from previous lesson
- Tape
- Eight flexible straws per student
- Scissors
- Centimeter rulers
- Centimeter graph paper – one for each student

Procedure

1. Place *Guess My Rule!* handout on an overhead projector.
2. Discuss each of the shapes and decide which shapes belong to the group. **Note:** only quadrilaterals belong in the group.
3. Challenge students to use flexible straws to make two quadrilaterals that look different. Review criteria for the shapes to be made: four sides, closed figure, no arcs, two-dimensional. Students should use only four straws for each shape.

If they wish, they can trim the straws to different lengths, but they should trim the shorter ends to about one-half inch and keep the bends to form the figures' vertices. The end of one straw can be inserted into the end of another.

4. Students can work with a partner and take turns making shapes and checking each other's work for accuracy.

5. Instruct the students to use theirrulers to trace both shapes (one on each side of a piece of graph paper).

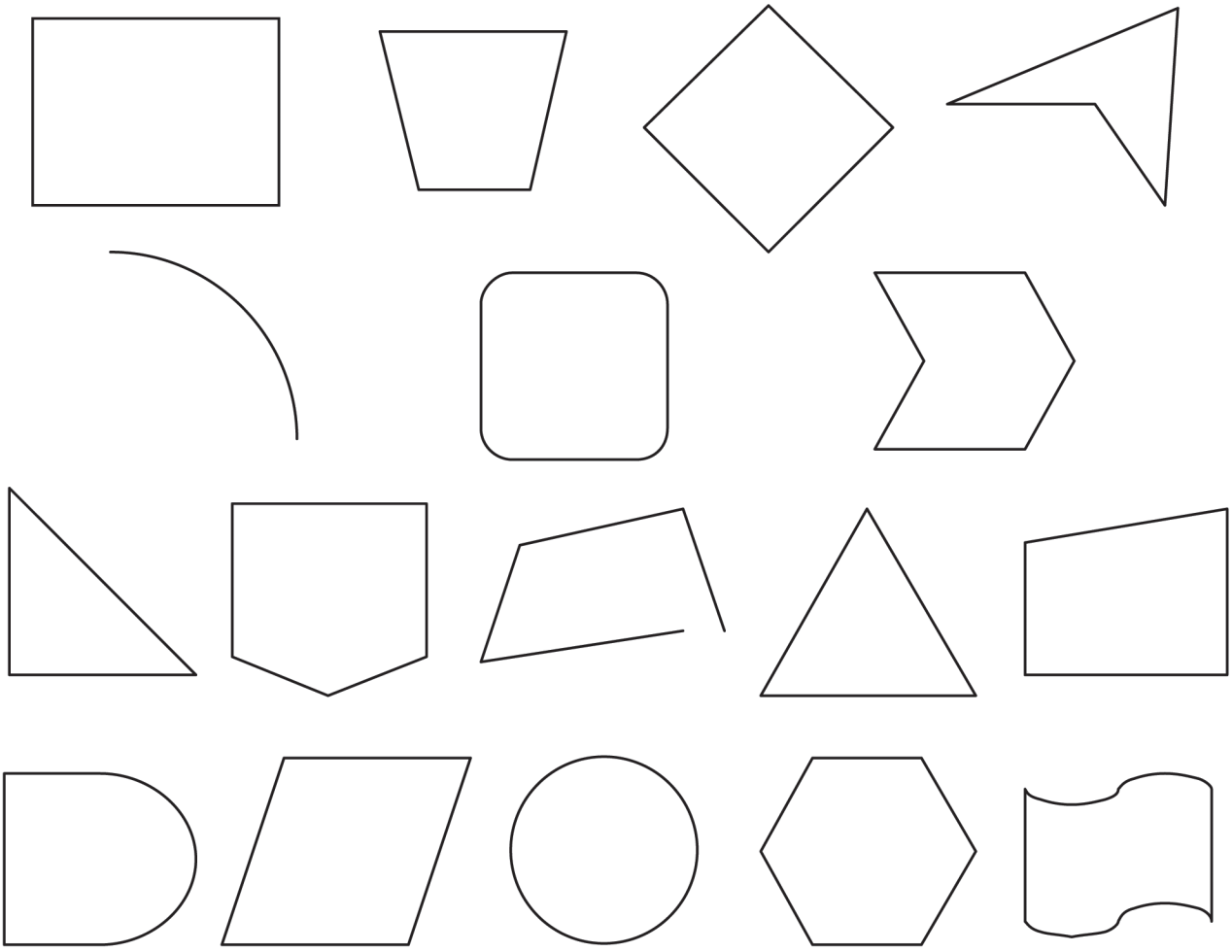
6. Discuss geometric vocabulary. The definitions of the following words were previously discussed and should be reviewed: congruent (sides and angles), vertex, vertices, angle, acute angle, right angle, obtuse angle and perpendicular.

7. Inside the shapes that the students have drawn on graph paper, list properties of each shape. Students should use vocabulary that they know. When asking students to list properties, questions such as "What do you see?" and "How do you know?" can encouragethem to analyze the figures.

8. In turn, students should share one of their shapes with the class. Guide students to think about the meaning of the word "parallel." Use class discussion to arrive at a simple definition and add the definition to the glossary. Include a drawing. Save the glossary for next lesson. See revised glossary at the end of the lesson.

9. Students should choose one of their shapes and cut it out. The shape should be saved for the next lesson.

Guess My Rule!



The shapes in the first row belong in the same group. The shapes in the second row do not belong.

Guess the rule!

Which of the other 10 shapes belong in the first row group?

Glossary

Acute angle: an angle that measures less than 90 degrees

Acute triangle: a triangle with no angle measuring 90 degrees or more

Angle: two rays that share an endpoint

Congruent: having exactly the same size and shape

Equilateral triangle: a triangle with three congruent sides

Isosceles triangle: a triangle with two congruent sides

Obtuse angle: an angle that measures greater than 90 degrees

Obtuse triangle: a triangle whose largest angle measures greater than 90 degrees

Parallel: always the same distance apart

Perpendicular: forming right angles

Polygon: a closed figure formed from line segments that meet only at their endpoints

Right angle: an angle that measures 90 degrees

Right triangle: a triangle with a right angle

Scalene triangle: a triangle with no congruent sides

Side: a line segment connected to other segments to form a polygon

Triangle: a three-sided polygon

Vertex: the point at which two line segments, lines, or rays meet to form an angle

Vertices: plural of vertex

Reference: Math on Call, A Mathematics Handbook

Lesson 4: Sorting Quadrilaterals

Overview

Students will analyze the properties of quadrilaterals and will use appropriate vocabulary to describe and classify quadrilaterals into groups and subgroups.

Benchmarks and Standards

Measurement

Grade 6

05-07 Benchmark

D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.

Y2003.CMA.S03.G05-07.BD.L06.I01 Characteristics and Properties /

01. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.

Materials

- Students' cut-out shape from previous lesson
- Glossary from previous lesson
- Masking tape
- Graph paper

Procedure

1. Invite six to eight students to tape their cut-out shape to the board. Review the criteria used for making these shapes: four sides, closed figure, no arcs, two-dimensional. Explain that all of these shapes are polygons. Ask, "What are all four-sided polygons called?"

2. Review vocabulary and add "quadrilateral" to the list.

3. Through class discussion, have students sort the shapes on the board into groups. Ask them to verbalize how they are sorting shapes. Questions such as the following can help students sort the shapes:
    - "What do you see?"
    - "How are the shapes alike and how are they different?"
    - "What properties written on each shape do all the shapes in that groups have in common?"

4. As students are sorting the shapes into categories, discuss the common properties of the shapes in each group. If it is possible for the students to do so, have them label the categories with the correct labels such rectangle, square, parallelogram, rhombus and trapezoid.

5. Either in groups or individually, have remaining students bring their shapes to the board. Use class discussion to determine into which group each shape should be sorted.

6. Ask students to look at the groups on the board. Ask: "Can any groups be sorted into subgroups? Can these subgroups be labeled?" **Note:** At this point of the lesson, the goal is to have the following groups: quadrilaterals, parallelograms, rectangles, squares, rhombuses and trapezoids. Students may need to be guided to reach this point.

7. Assign one student to each group of shapes. Ask the students to write the common properties of the shapes in the group. Students should use appropriate vocabulary and can solicit help from the other students in the class. Allow students to refer to the glossary. This work should be saved for the next lesson.

8. Remove shapes from the board, paper clip together the shapes belonging to each group to the paper with the properties defining them and save for the next lesson.

Glossary

Acute angle: an angle that measures less than 90 degrees

Acute triangle: a triangle with no angle measuring 90 degrees or more

Angle: two rays that share an endpoint

Congruent: having exactly the same size and shape

Equilateral triangle: a triangle with three congruent sides

Isosceles triangle: a triangle with two congruent sides

Obtuse angle: an angle that measures greater than 90 degrees

Obtuse triangle: a triangle whose largest angle measures greater than 90 degrees

Parallel: always the same distance apart

Perpendicular: forming right angles

Polygon: a closed figure formed from line segments that meet only at their endpoints

Quadrilateral: a four-sided polygon

Right angle: an angle that measures 90 degrees

Right triangle: a triangle with a right angle

Scalene triangle: a triangle with no congruent sides

Side: a line segment connected to other segments to form a polygon

Triangle: a three-sided polygon

Vertex: the point at which two line segments, lines, or rays meet to form an angle

Vertices: plural of vertex

Reference: Math on Call, A Mathematics Handbook

Lesson 5: Organizing Quadrilaterals

A Quadrilateral Family Tree

Overview

Students will analyze the properties of quadrilaterals.

Benchmarks and Standards

Measurement

Grade 6

05-07 Benchmark

D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.

Y2003.CMA.S03.G05-07.BD.L06.I01 Characteristics and Properties /

01. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.

Y2003.CMA.S03.G05-07.BD.L06.I02 Characteristics and Properties /

02. Use standard language to define geometric vocabulary: vertex, face, altitude, diagonal, isosceles, equilateral, acute, obtuse, and other vocabulary as appropriate.

Materials

- Papers with lists of properties and students' shapes from previous lesson
- Glossary from previous lesson
- Masking tape
- Graph paper
- Examples of all types of quadrilaterals and their properties
- *A Quadrilateral Family Tree* student handout

Procedure

1. From previous lesson, display sheets of paper with list of properties.

2. Next to each sheet, tape shapes that are representative of the list of properties.

3. Review the vocabulary in the lists of properties.

4. Ask students to look at shapes and determine whether any of them share properties. Ask: "Do parallelograms and rectangles have any properties in common?" and "How are rectangles and parallelograms alike and different?"

5. Help students think about properties that progressively limit the types of shapes that can be in the group. For example, all the shapes are quadrilaterals, but only some of them are parallelograms. In the group of parallelograms, all the shapes have two pairs of parallel sides, but only some of them have four right angles. Allow students to take turns moving the shapes around on the board.

6. If there are no examples of trapezoids, then introduce them. List the properties. Two subgroups of trapezoids are right trapezoids and isosceles trapezoids. Introduce these shapes and their properties. Ask: "How are trapezoids alike and different from the other quadrilaterals?" and "Why do you think we call this shape an isosceles trapezoid? A right trapezoid?"

7. On the board, replace shapes and lists of properties with the materials included in this lesson. Explain that the properties listed are the properties needed to distinguish one type of shape from another. Students should think about how to classify and organize the shapes in such a way that groups and subgroups are arranged into a logical hierarchy. Again, allow students to take turns moving the shapes around on the board.

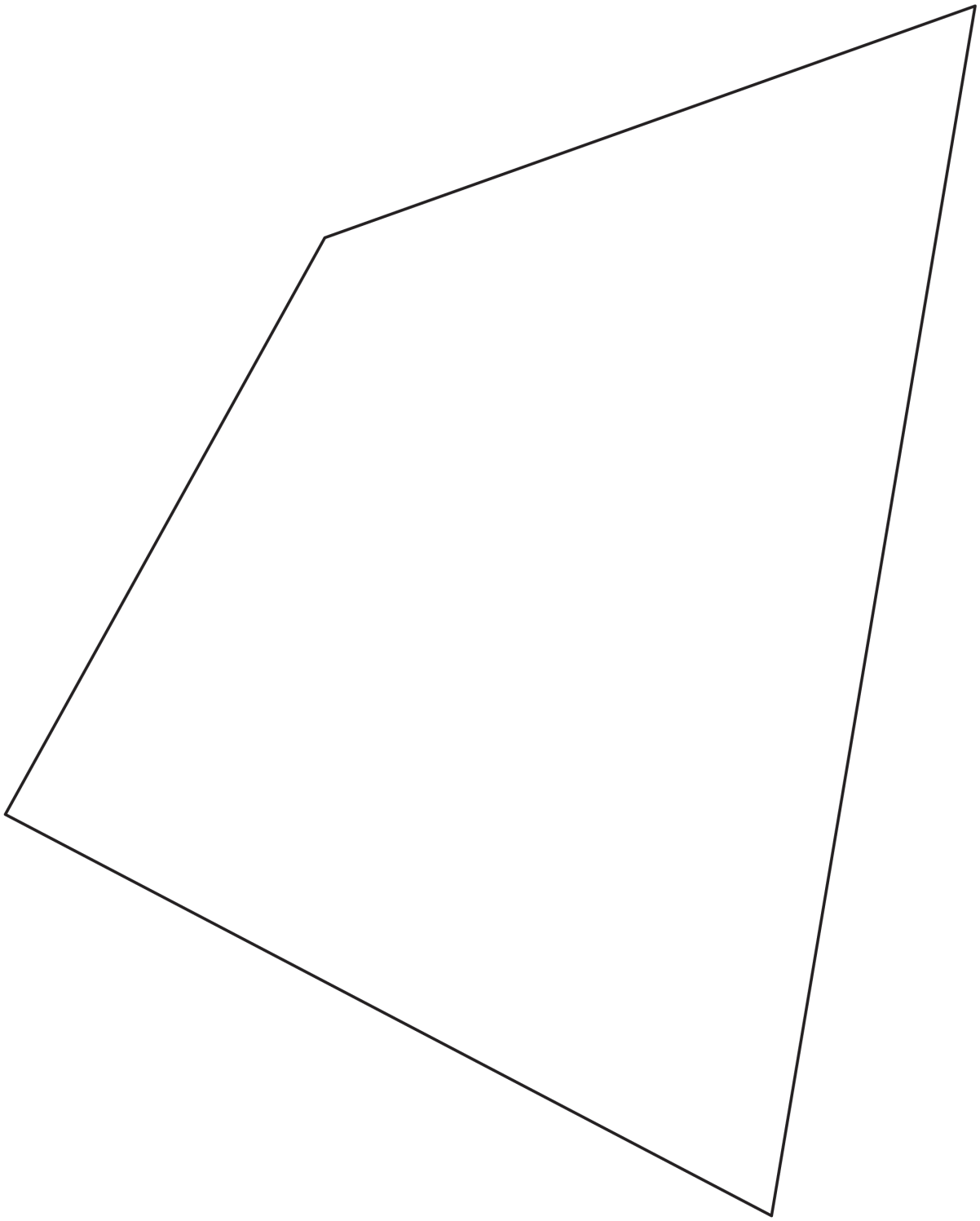
8. Students may need to be guided to organize the shapes into a quadrilateral family tree similar to the one provided in this lesson. Once the tree is organized, discuss the arrangement of the shapes.

9. Students should copy the shapes, the properties of the shapes and each shape's position on the tree.

10. The names (labels) of these shapes and their properties have been added to the glossary.

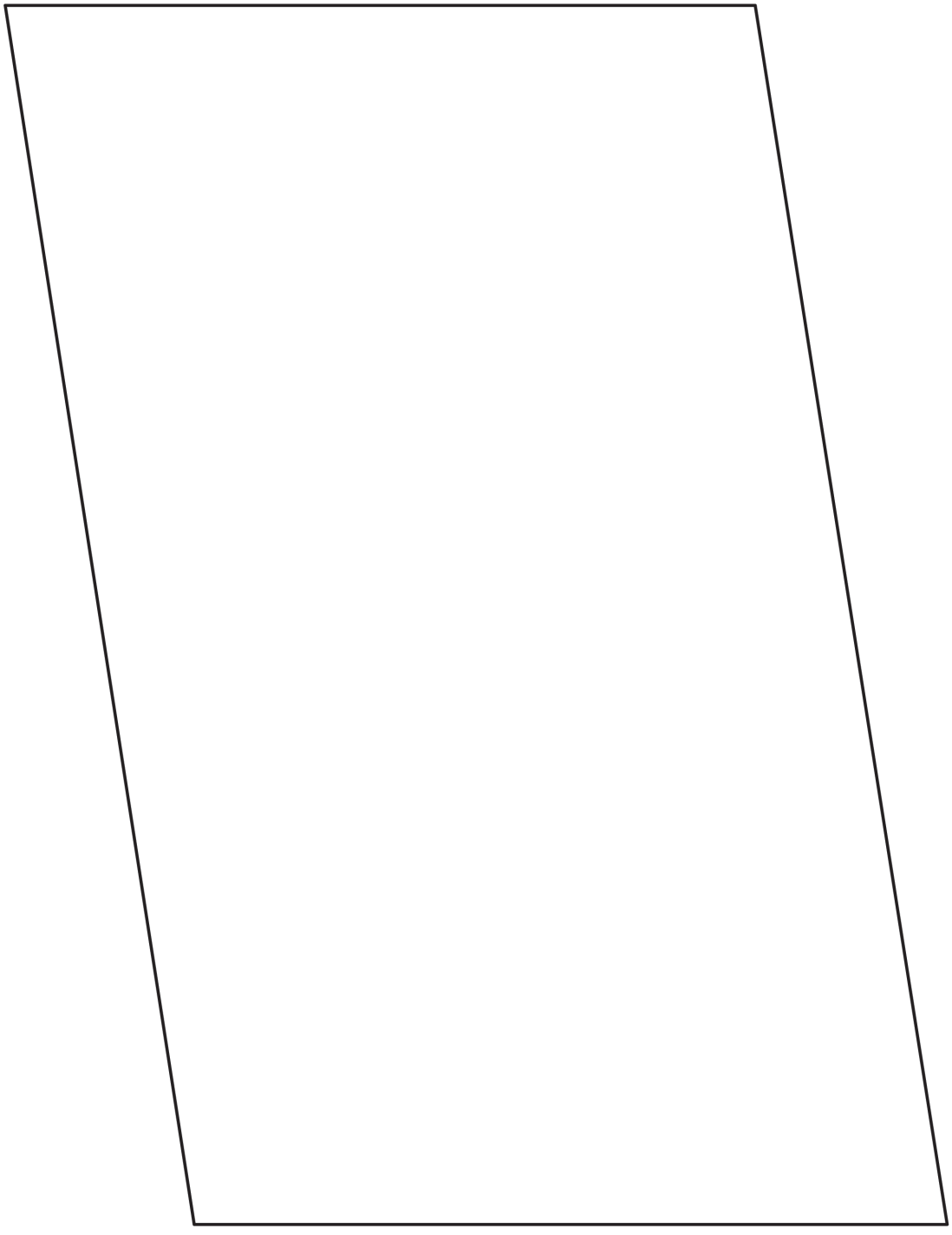
Quadrilateral

- A four-sided polygon



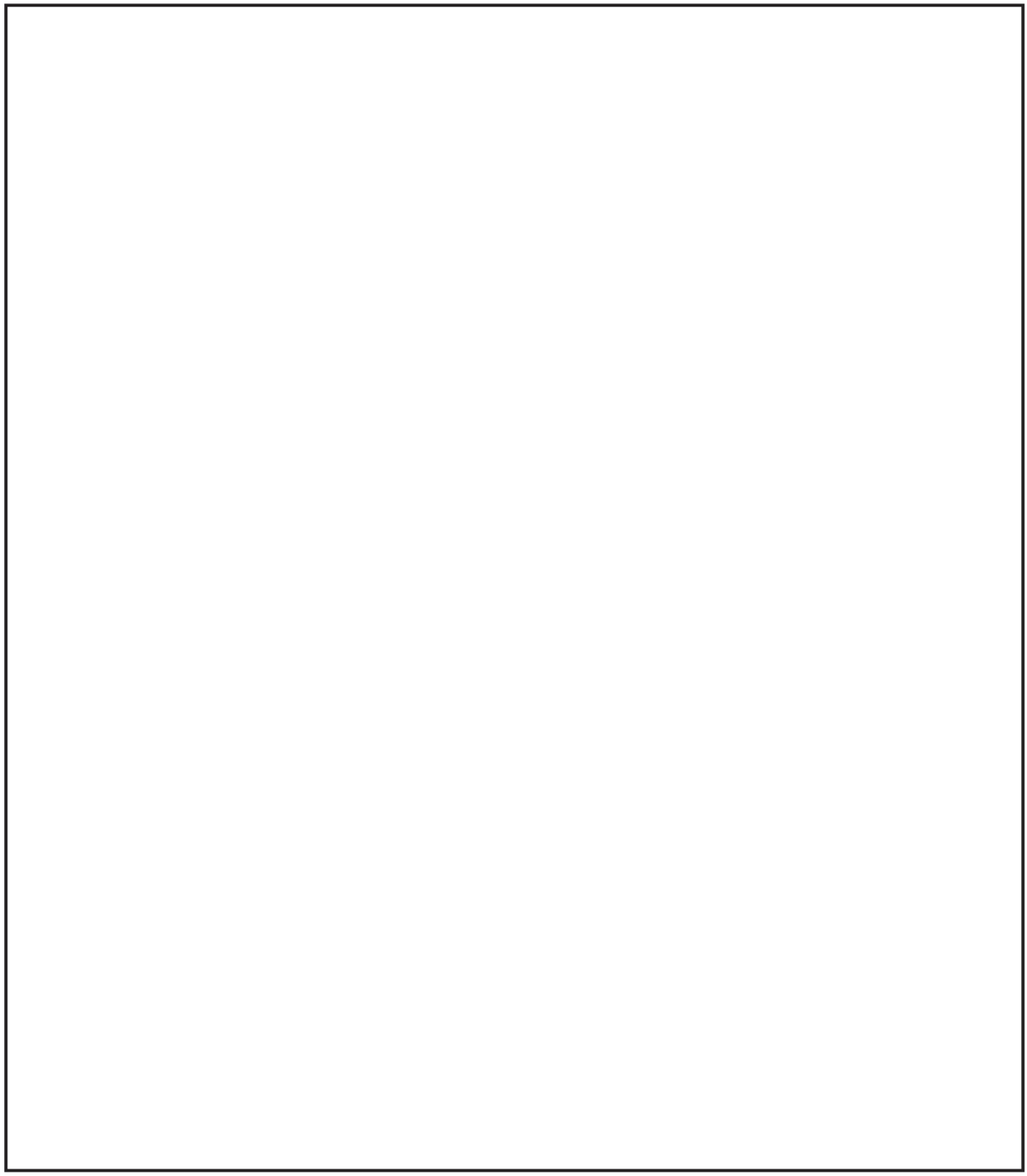
Parallelogram

- Two pairs of parallel and congruent sides
- Opposite angles are congruent



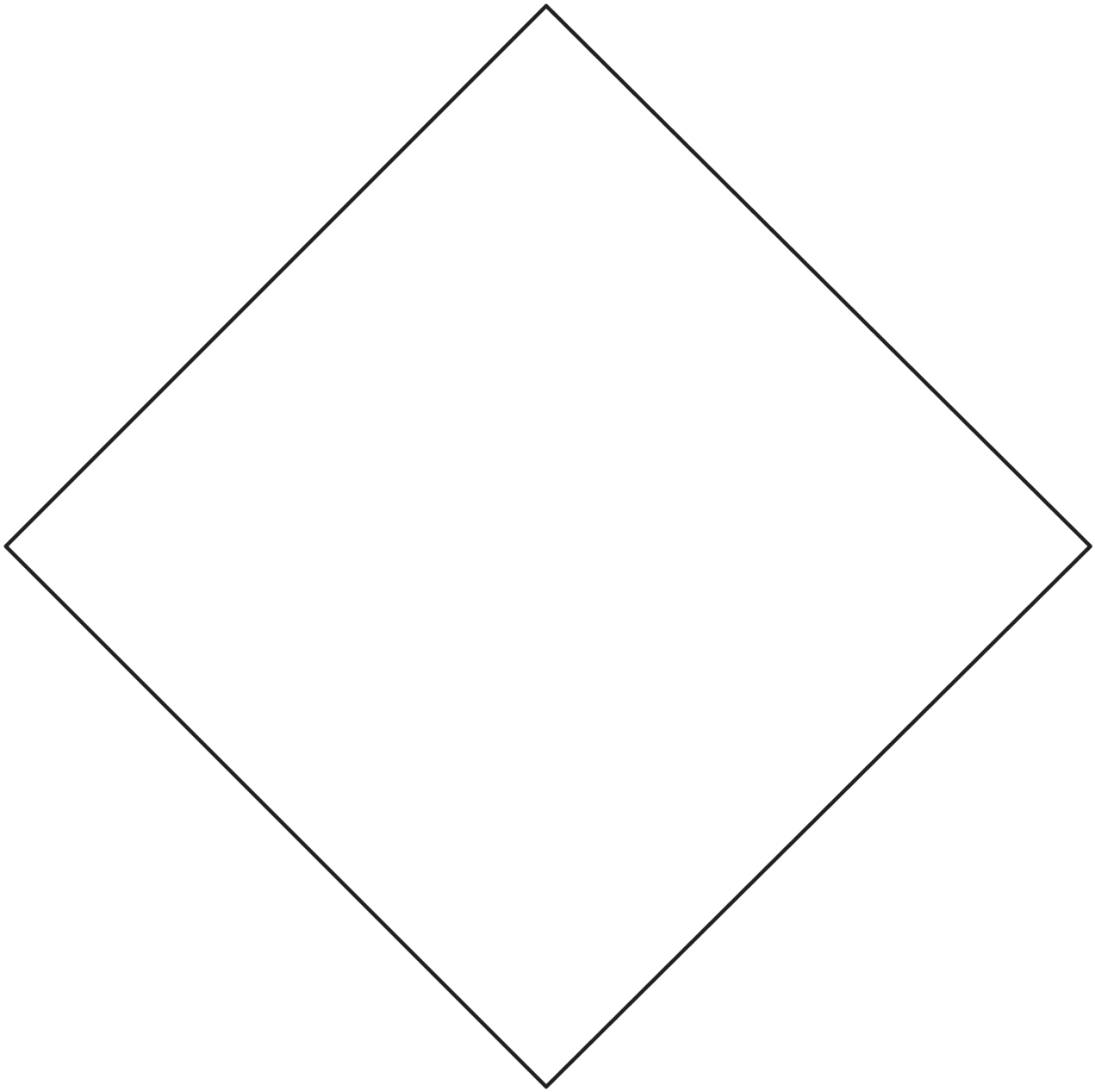
Rectangle

- Two pairs of parallel and congruent sides
- Four right angles



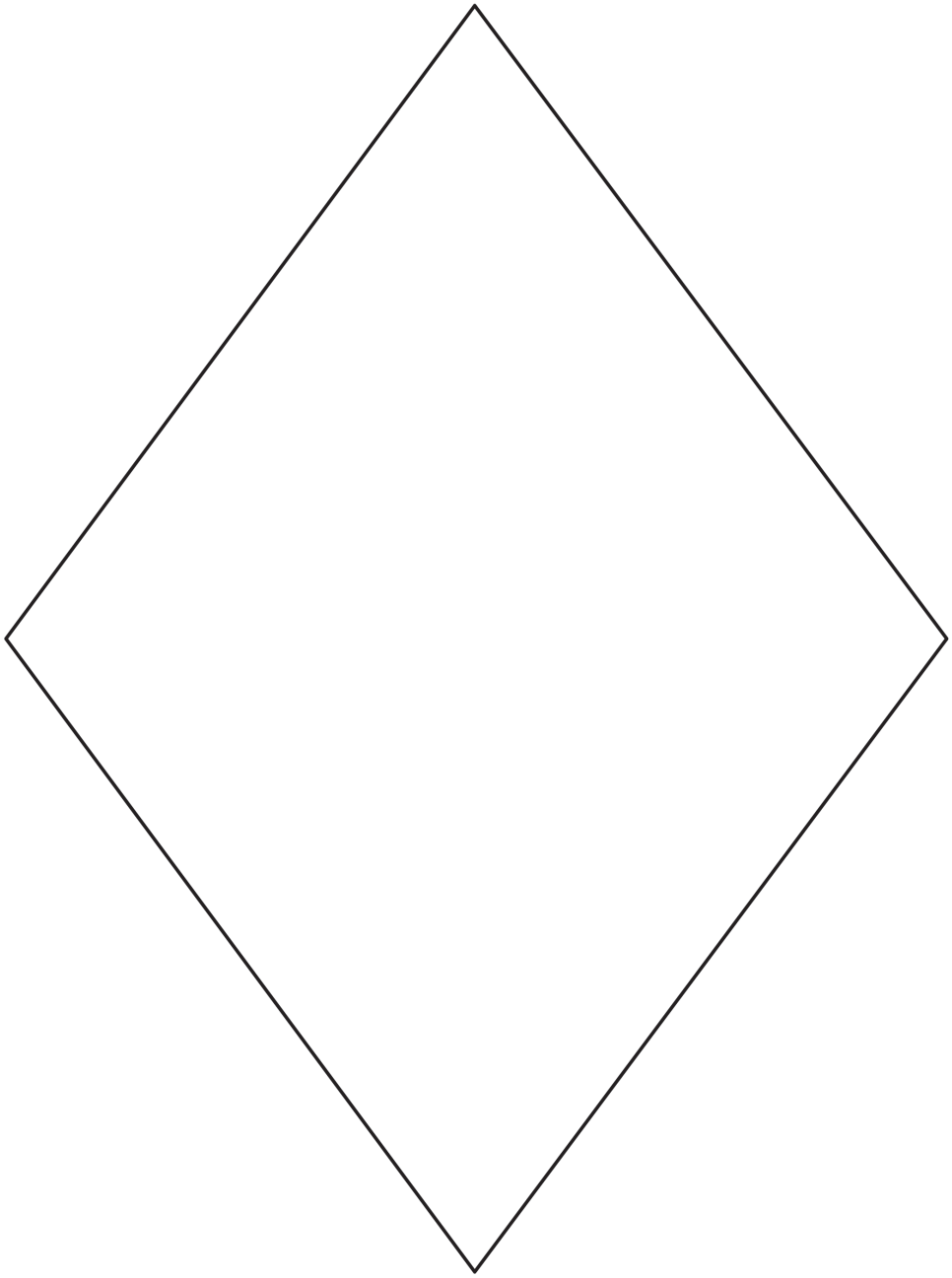
Square

- Two pairs of parallel sides
- Four congruent sides
- Four right angles



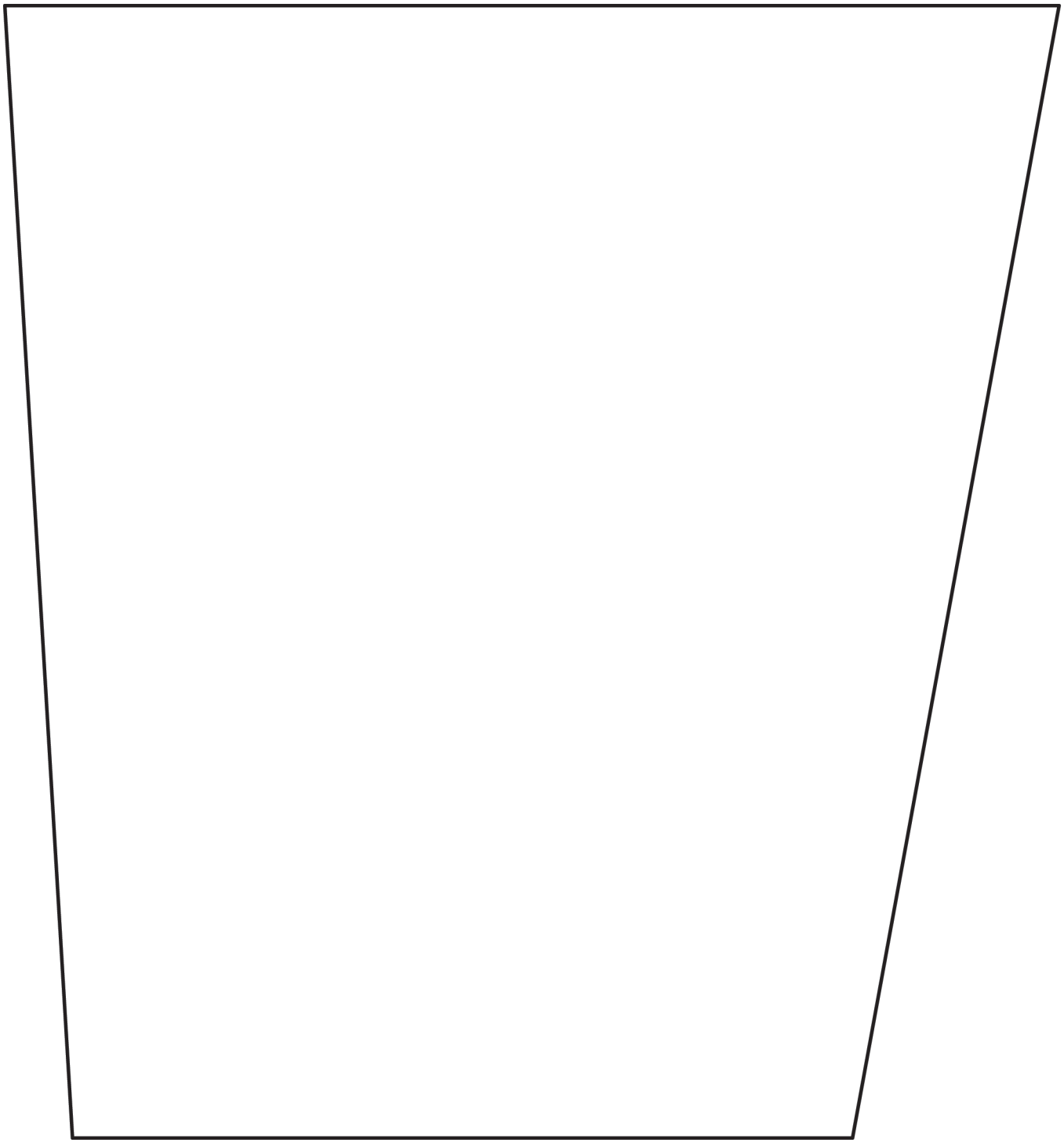
Rhombus

- Two pairs of parallel sides
- Four congruent sides
- Opposite angles are congruent



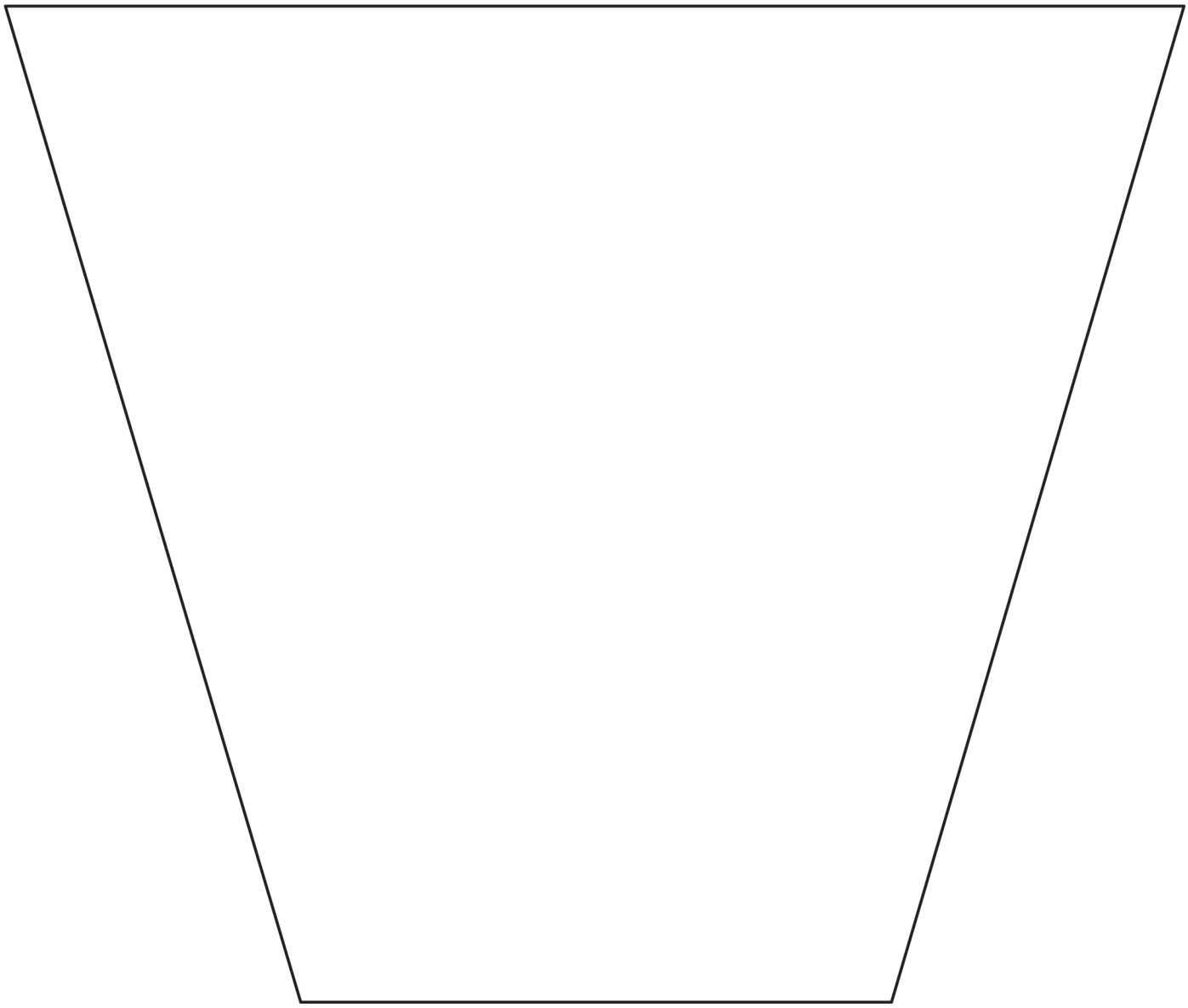
Trapezoid

- Exactly one pair of parallel sides



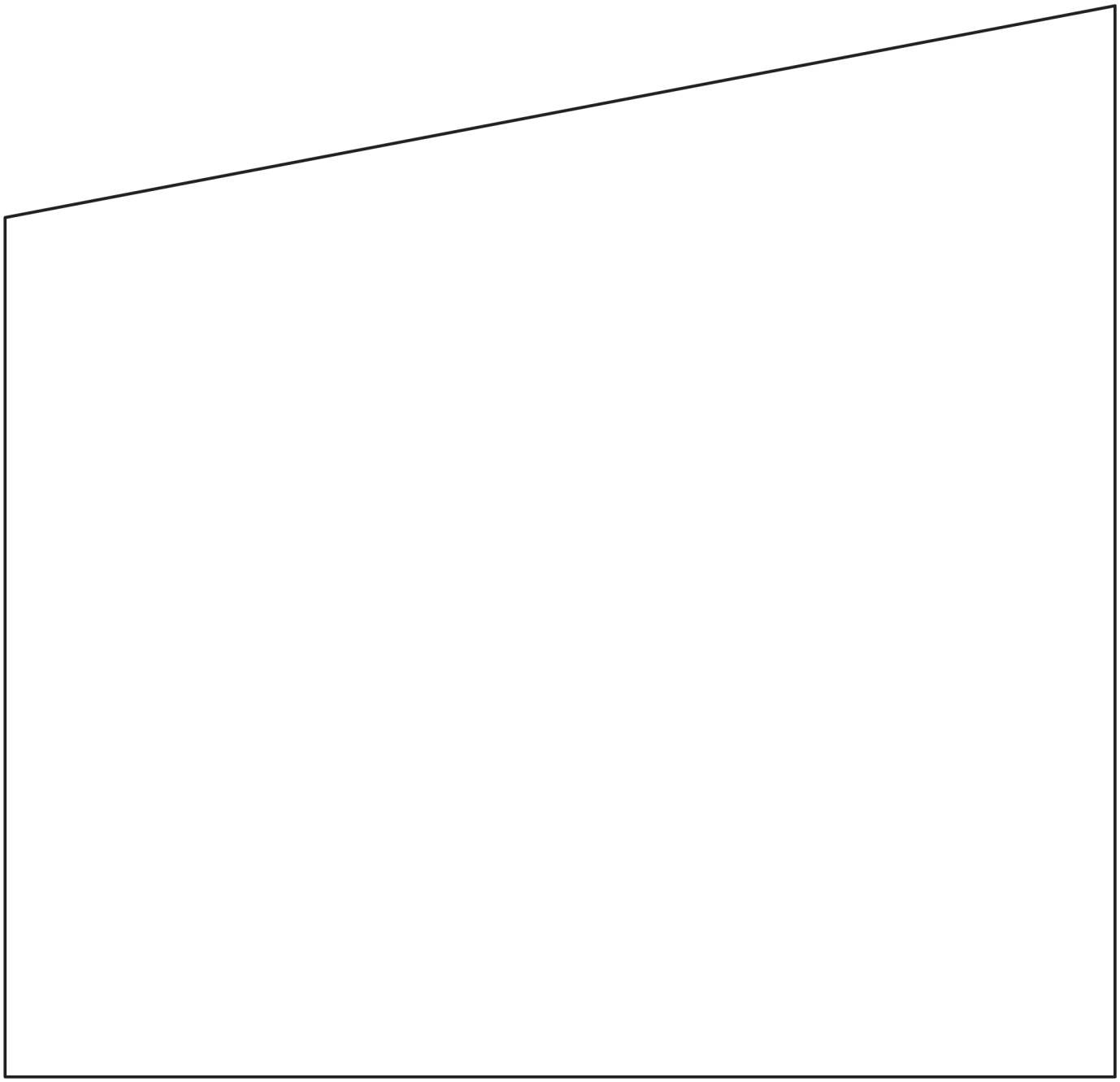
Isosceles Trapezoid

- Exactly one pair of parallel sides
- Two pairs of adjacent, congruent angles

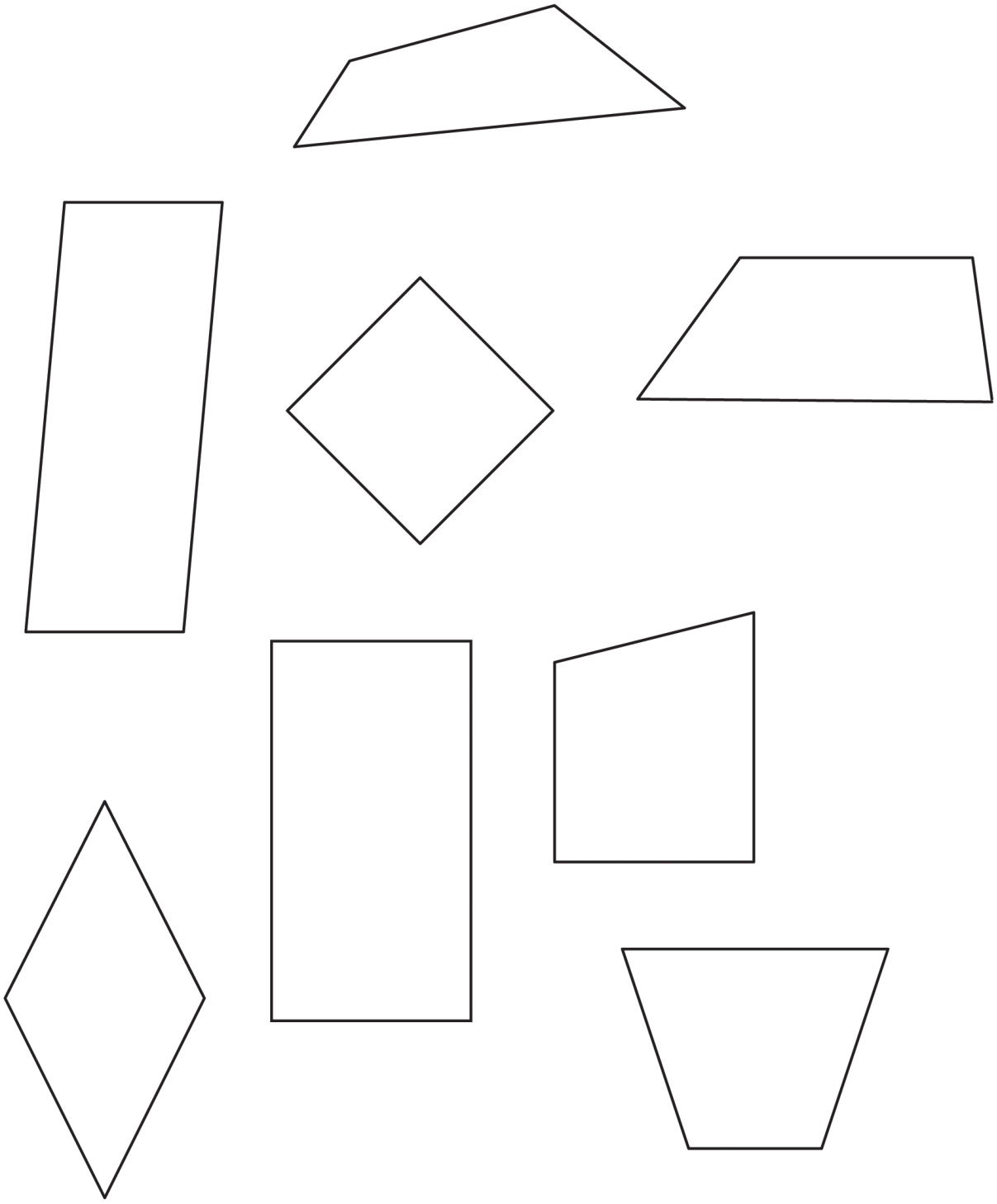


Right Trapezoid

- Exactly one pair of parallel sides
- Two adjacent right angles



A Quadrilateral Family Tree



Glossary

Acute angle: an angle that measures less than 90 degrees

Acute triangle: a triangle with no angle measuring 90 degrees or more

Angle: two rays that share an endpoint

Congruent: having exactly the same size and shape

Equilateral triangle: a triangle with three congruent sides

Isosceles trapezoid: a trapezoid with two pairs of adjacent and congruent angles

Isosceles triangle: a triangle with two congruent sides

Obtuse angle: an angle that measures greater than 90 degrees

Obtuse triangle: a triangle whose largest angle measures greater than 90 degrees

Parallel: always the same distance apart

Parallelogram: a quadrilateral with two pairs of parallel and congruent sides; opposite angles are congruent

Perpendicular: forming right angles

Polygon: a closed figure formed from line segments that meet only at their endpoints

Quadrilateral: a four-sided polygon

Rectangle: a quadrilateral with two pairs of parallel and congruent sides and four right angles

Rhombus: a quadrilateral with two pairs of parallel sides and four congruent sides; opposite angles are congruent

Right angle: an angle that measures 90 degrees

Right trapezoid: a trapezoid with two adjacent right angles

Right triangle: a triangle with a right angle

Scalene triangle: a triangle with no congruent sides

Side: a line segment connected to other segments to form a polygon

Square: a quadrilateral with two pairs of parallel sides, four congruent sides and four right angles

Trapezoid: a quadrilateral with exactly one pair of parallel sides

Triangle: a three-sided polygon

Vertex: the point at which two line segments, lines, or rays meet to form an angle

Vertices: plural of vertex

Reference: Math on Call, A Mathematics Handbook

Lesson 6: Analyzing the Quadrilateral Family Tree

Using "All" and "Some"

Overview

Students will analyze the properties of quadrilaterals and use their understanding of those properties to classify shapes.

Benchmarks and Standards

Measurement

Grade 6

05-07 Benchmark

D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.

Y2003.CMA.S03.G05-07.BD.L06.I01 Characteristics and Properties /

01. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.

Y2003.CMA.S03.G05-07.BD.L06.I02 Characteristics and Properties /

02. Use standard language to define geometric vocabulary: vertex, face, altitude, diagonal, isosceles, equilateral, acute, obtuse, and other vocabulary as appropriate.

Materials

- Students' quadrilateral family trees
- Glossary from previous lesson
- Masking tape
- Sticky notes
- From previous lesson, examples of types of quadrilaterals and their properties
- Examples of types of quadrilaterals and their properties, and a list of each shape's names
- Edited glossary that includes a list of each shape's names

Procedure

1. From the previous lesson, tape examples of types of quadrilaterals and their properties to the chalkboard. Arrange the quadrilaterals as a family tree.

2. Facilitate a class discussion. Ask students to look at shapes and, on sticky notes, list all the ways that they can label or name each shape. Ask students to consider the properties that are unique to each shape. Your questions may include the following:

    - Are all the shapes quadrilaterals? Polygons? How do you know?
    - Are rectangles parallelograms? How can you decide?
    - Can a square be labeled as a rectangle? As a parallelogram? How do you know?

3. The objective is to find as many labels or names for each shape as possible. Students should write labels on sticky notes and put them next to the appropriate quadrilaterals on the board.

4. After students have generated a number of labels, facilitate another class discussion. Ask them to use their understanding of the properties of the shapes to write statements about the quadrilaterals using the words "all" and "some." Students should use geometric vocabulary and justify their statements. Encourage them to use the organization of the quadrilateral family tree to help them compose their statements. Ask them to think about the properties associated with each type of quadrilateral. Questions such as "What makes a square a square?" can be helpful in stimulating students' thinking.

    Some examples of statements include the following:

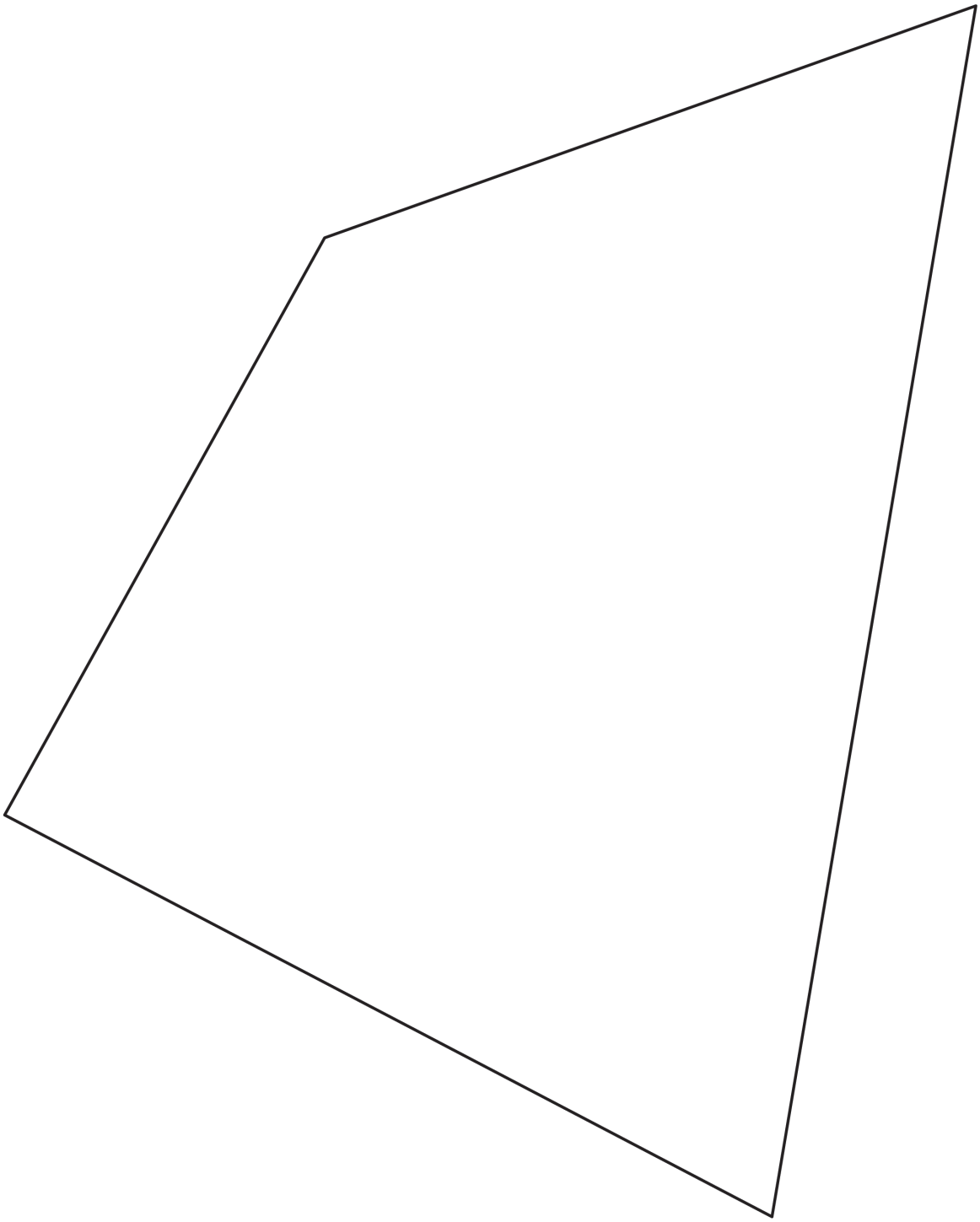
    - All rectangles are quadrilaterals, but only some quadrilaterals are rectangles. All rectangles are quadrilaterals because all rectangles have four sides. Only some quadrilaterals are rectangles because rectangles have two pairs of parallel sides and four right angles and not all quadrilaterals have those properties.
    - All rhombuses are parallelograms, but only some parallelograms are rhombuses. All rhombuses and parallelograms have two pairs of parallel sides and opposite angles that are congruent, but rhombuses have four congruent sides and only some parallelograms have four congruent sides.

5. On the board, replace shapes and lists of properties with the shapes included in this lesson. Discuss the properties of these shapes. Do students agree or disagree with the lists of properties?

6. Based on the information discussed, students can edit their quadrilateral family trees.

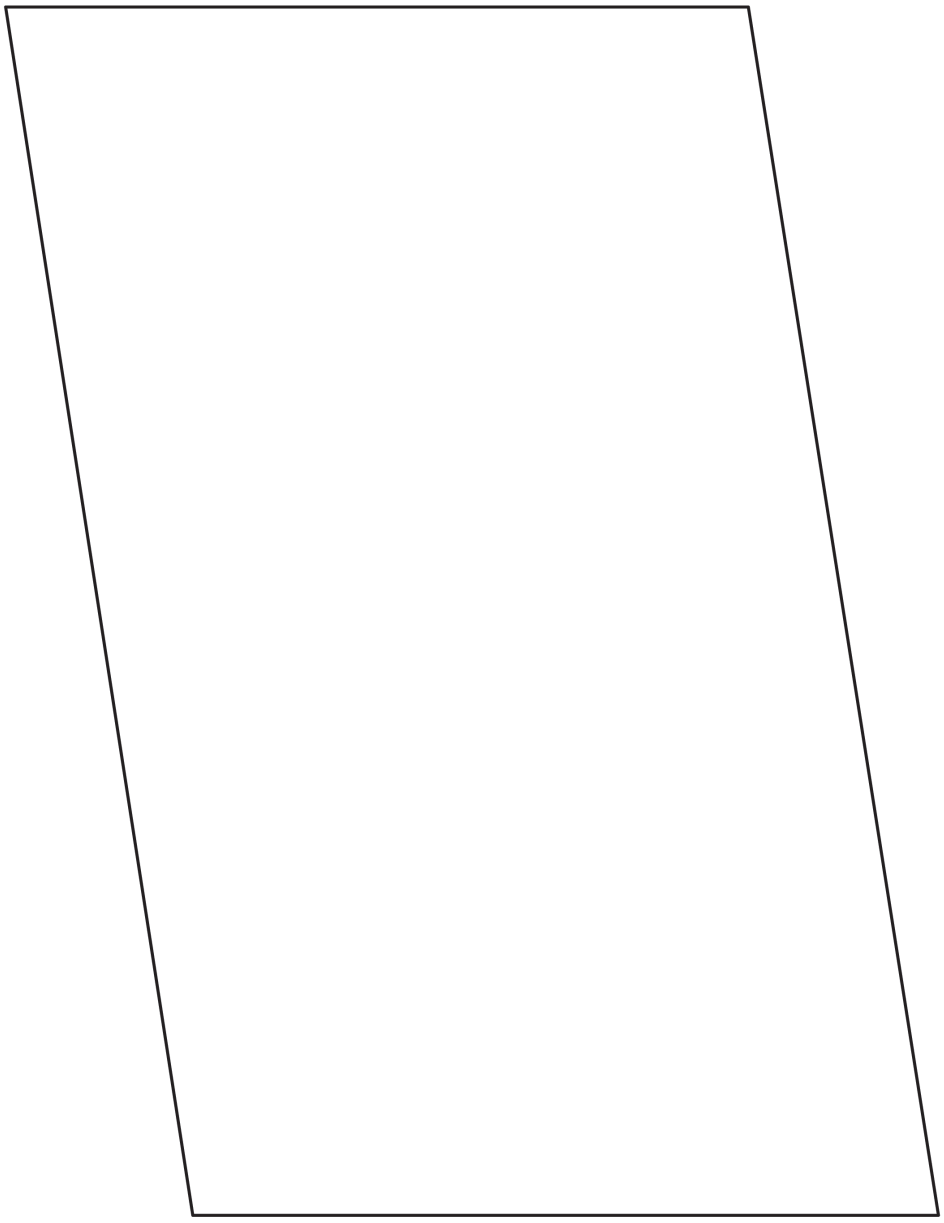
Quadrilateral

- A four-sided polygon



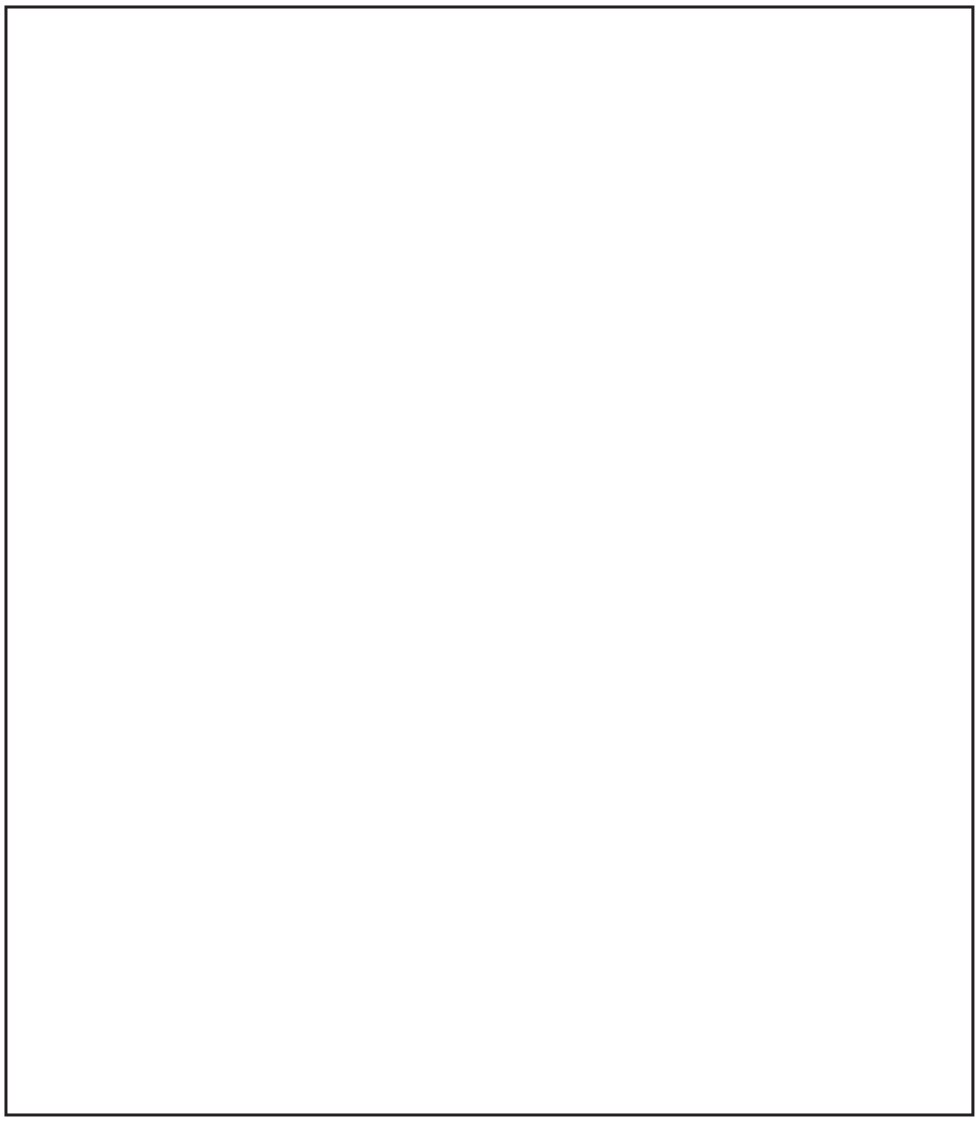
Parallelogram

- A polygon
- A quadrilateral
- Two pairs of parallel and congruent sides
- Opposite angles are congruent



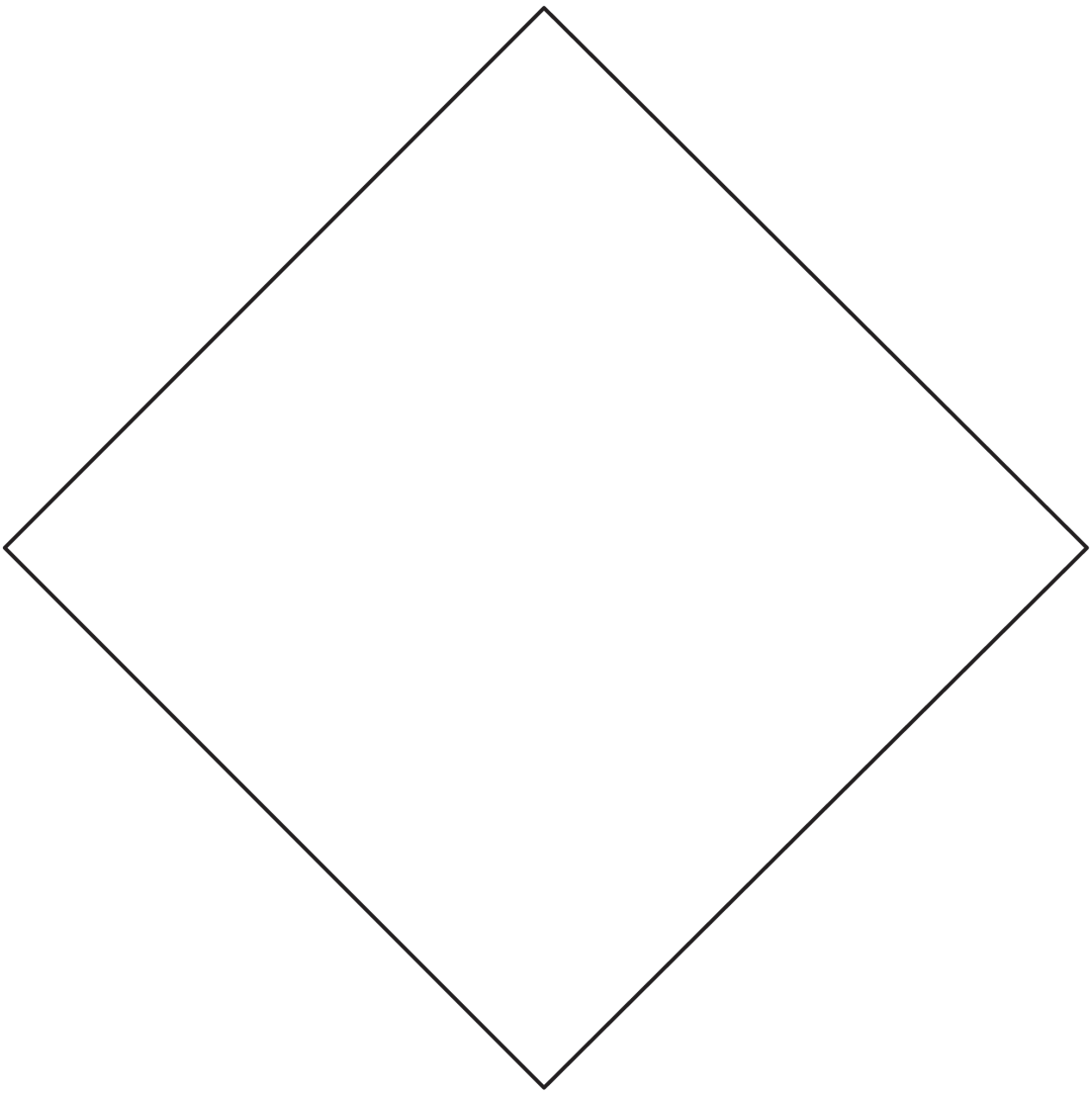
Rectangle

- A polygon
- A quadrilateral
- A parallelogram
- Two pairs of parallel and congruent sides
- Four right angles



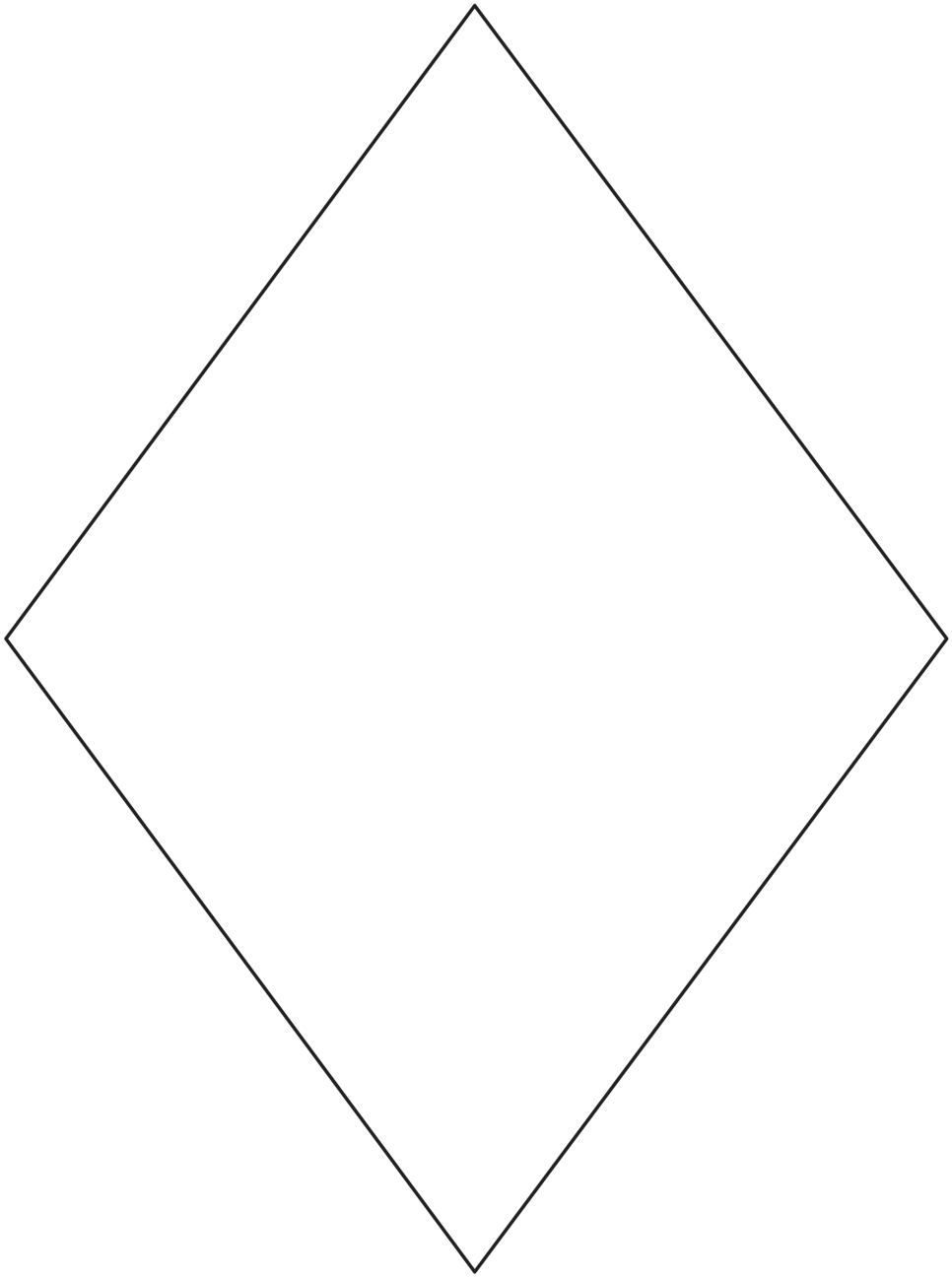
Square

- A polygon
- A quadrilateral
- A parallelogram
- A rectangle
- A rhombus
- Two pairs of parallel sides
- Four congruent sides
- Four right angles



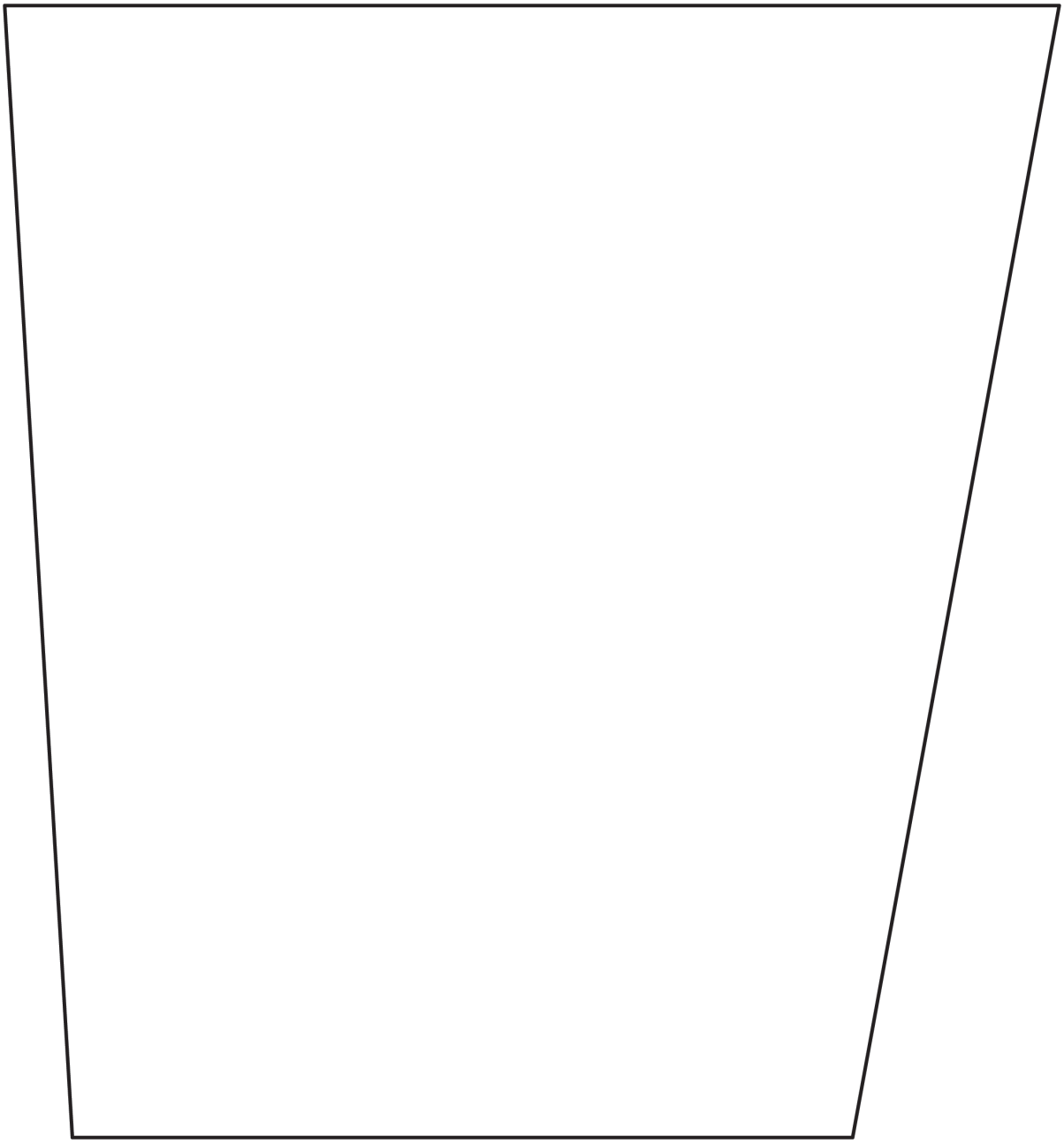
Rhombus

- A polygon
- A quadrilateral
- A parallelogram
- Two pairs of parallel sides
- Four congruent sides
- Opposite angles are congruent



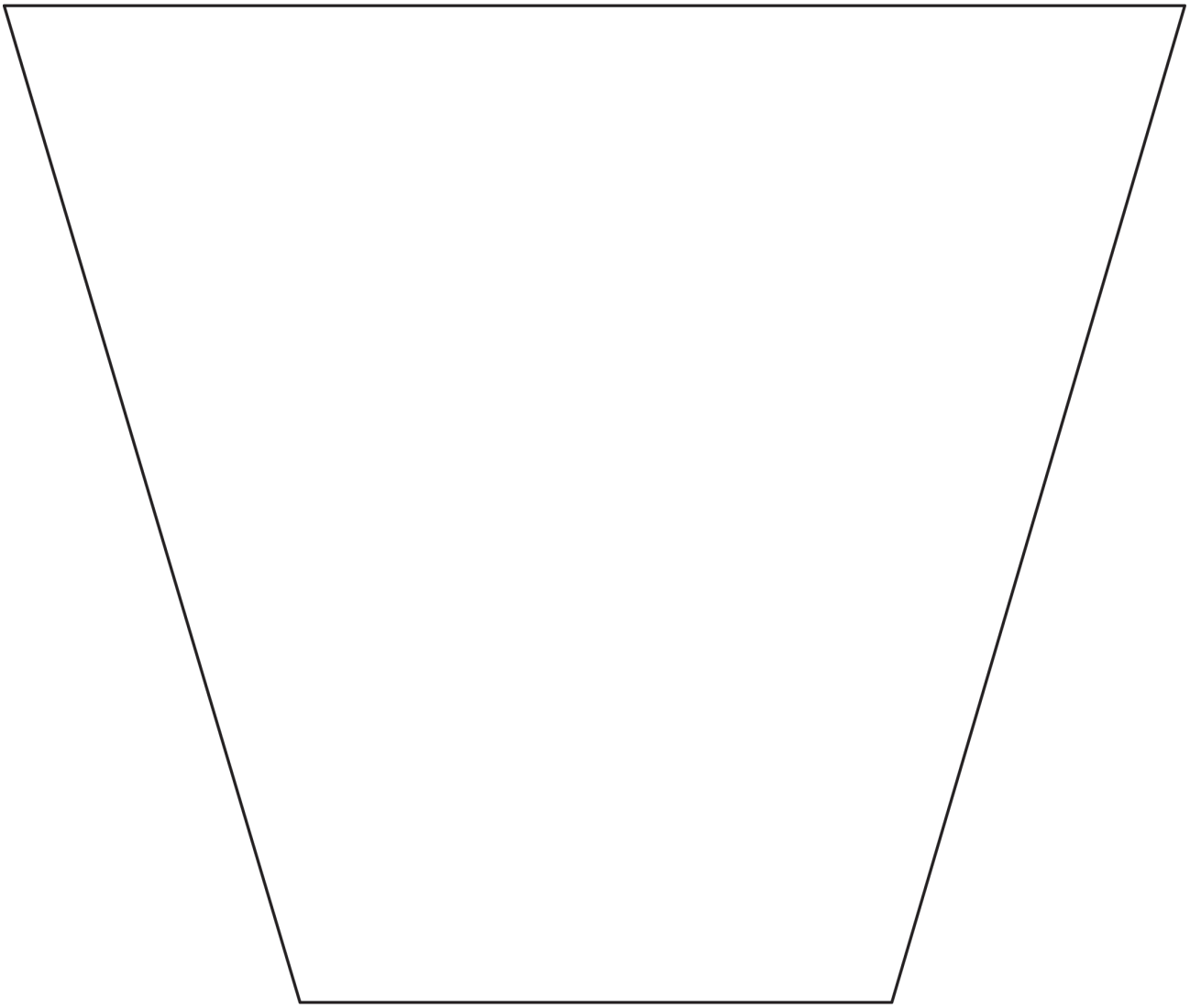
Trapezoid

- A polygon
- A quadrilateral
- Exactly one pair of parallel sides



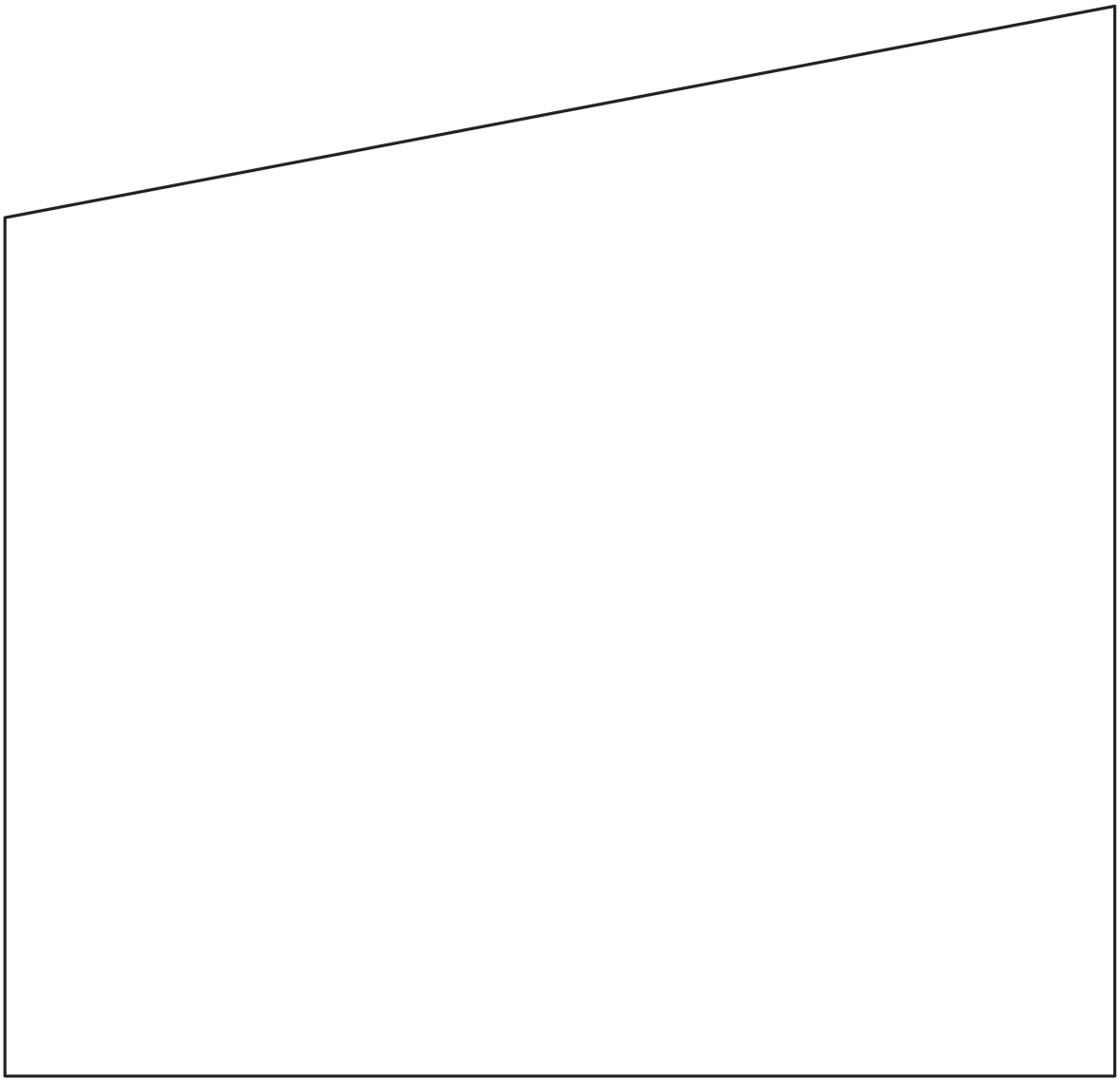
Isosceles Trapezoid

- A polygon
- A quadrilateral
- A trapezoid
- Exactly one pair of parallel sides
- Two pairs of adjacent, congruent angles

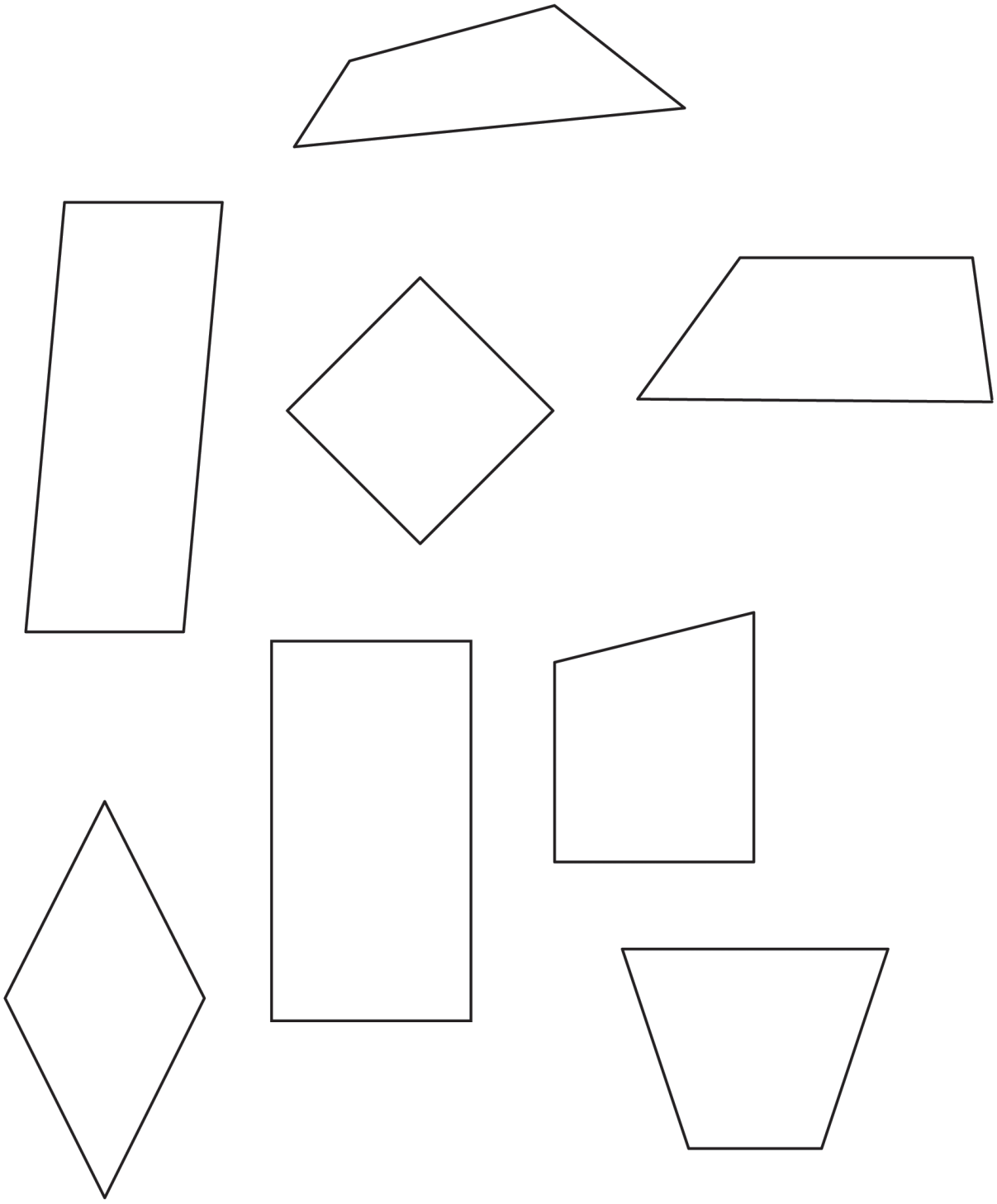


Right Trapezoid

- A polygon
- A quadrilateral
- A trapezoid
- Exactly one pair of parallel sides
- Two adjacent right angles



A Quadrilateral Family Tree



Glossary

Acute angle: an angle that measures less than 90 degrees

Acute triangle: a triangle with no angle measuring 90 degrees or more

Angle: two rays that share an endpoint

Congruent: having exactly the same size and shape

Equilateral triangle: a triangle with three congruent sides

Isosceles trapezoid: a trapezoid with two pairs of adjacent and congruent angles. An isosceles trapezoid is also a polygon and a quadrilateral.

Isosceles triangle: a triangle with two congruent sides

Obtuse angle: an angle that measures greater than 90 degrees

Obtuse triangle: a triangle whose largest angle measures greater than 90 degrees

Parallel: always the same distance apart

Parallelogram: a quadrilateral with two pairs of parallel and congruent sides; opposite angles are congruent. A parallelogram is also a polygon.

Perpendicular: forming right angles

Polygon: a closed figure formed from line segments that meet only at their endpoints

Quadrilateral: a four-sided polygon

Rectangle: a quadrilateral with two pairs of parallel and congruent sides and four right angles. A rectangle is also a polygon and a parallelogram.

Rhombus: a quadrilateral with two pairs of parallel sides and four congruent sides; opposite angles are congruent. A rhombus is also a polygon and a parallelogram.

Right angle: an angle that measures 90 degrees

Right trapezoid: a trapezoid with two adjacent right angles. A right trapezoid is also a polygon and a quadrilateral.

Right triangle: a triangle with a right angle

Scalene triangle: a triangle with no congruent sides

Side: a line segment connected to other segments to form a polygon

Square: a quadrilateral with two pairs of parallel sides, four congruent sides and four right angles. A square is also a polygon, a parallelogram, a rectangle, and a rhombus.

Trapezoid: a quadrilateral with exactly one pair of parallel sides. A trapezoid is also a polygon.

Triangle: a three-sided polygon

Vertex: the point at which two line segments, lines, or rays meet to form an angle

Vertices: plural of vertex

Reference: Math on Call, A Mathematics Handbook

Lesson 7: The Important Book

Overview

Using their understanding of the properties of triangles and quadrilaterals, the students will create a book.

Benchmarks and Standards

Measurement

Grade 6

05-07 Benchmark

D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.

Y2003.CMA.S03.G05-07.BD.L06.I01 Characteristics and Properties /

01. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.

Y2003.CMA.S03.G05-07.BD.L06.I02 Characteristics and Properties /

02. Use standard language to define geometric vocabulary: vertex, face, altitude, diagonal, isosceles, equilateral, acute, obtuse, and other vocabulary as appropriate.

Y2003.CMA.S02.G0507.BC.L06.I03 Use Measurement Techniques and Tools /

03. Estimate perimeter or circumference and area for circles, triangles and quadrilaterals, and surface area and volume for prisms and cylinders by:
    - Estimating lengths using string or links, areas using tiles or grid, and volumes using cubes;
    - Measuring attributes (diameter, side lengths or heights) and using established formulas for circles, triangles, rectangles, parallelograms and rectangular prisms.

Materials

- Students' quadrilateral family trees
- Copies of edited glossary from previous lessons (included)
- Typing paper
- A copy of "The Important Book" by Margaret Wise Brown (Harper Collins, publisher)

Procedure

1. Read "The Important Book" to the class.

2. Discuss the pattern of the language in the book.

3. Explain to students that they are going to use a technique called "copy change" to make a class book about triangles and quadrilaterals. Explain that students will use the pattern of the writing in"The Important Book" as a framework for creating their book. For example, in the book the text reads:

   "The important thing about a spoon is that you eat with it.

   It's like a little shovel,

   You hold it in your hand,

   You can put it in your mouth,

   It isn't flat,

   It's hollow,

   And it spoons things up.

   But the important thing about a spoon is that you eat with it."

   For the class book, students will substitute types of triangles and quadrilaterals in the text. The goal for this lesson is for students to determine what are the most important, or defining, properties of each shape. For example, the text about a rectangle might read:

   "The important thing about a rectangle is it has two pairs of parallel sides and four right angles.

   It has four congruent sides.

   It is a polygon.

   It is a quadrilateral.

   It is a parallelogram.

   It is a shape used for a door or a window.

   But the important thing about a rectangle is it has two pairs of parallel sides and four right angles."

4. The book can include any of the shapes covered in the previous lessons: triangle, equilateral triangle, isosceles triangle, scalene triangle, acute triangle, right triangle, obtuse triangle, polygon, quadrilateral, parallelogram, rectangle, square, rhombus, trapezoid, right trapezoid and isosceles trapezoid. The book can also include the glossary of terms developed during the lessons. The glossary can include the properties and labels (or names) for each shape or only those that are important.

5. This project can be as simple or complex as the teacher wishes to make it. The descriptions of the shapes can be typed or handwritten. Students can illustrate their shapes on the same page as the text or the book can be assembled first and the illustrations for each shape can be drawn on the back of the text for the previous shape. The pages, including a cover and a list of authors, can be stapled or bound.

6. Assign different shapes to individual students or groups of students. Students should have their copies of their quadrilateral family trees. Make copies of the glossary available to all students. Students who finish their pages early can work on illustrations, the glossary or the cover.

Glossary

Acute angle: an angle that measures less than 90 degrees

Acute triangle: a triangle with no angle measuring 90 degrees or more

Angle: two rays that share an endpoint

Congruent: having exactly the same size and shape

Equilateral triangle: a triangle with three congruent sides

Isosceles trapezoid: a trapezoid with two pairs of adjacent and congruent angles. An isosceles trapezoid is also a polygon and a quadrilateral.

Isosceles triangle: a triangle with two congruent sides

Obtuse angle: an angle that measures greater than 90 degrees

Obtuse triangle: a triangle whose largest angle measures greater than 90 degrees

Parallel: always the same distance apart

Parallelogram: a quadrilateral with two pairs of parallel and congruent sides; opposite angles are congruent. A parallelogram is also a polygon.

Perpendicular: forming right angles

Polygon: a closed figure formed from line segments that meet only at their endpoints

Quadrilateral: a four-sided polygon

Rectangle: a quadrilateral with two pairs of parallel and congruent sides and four right angles. A rectangle is also a polygon and a parallelogram.

Rhombus: a quadrilateral with two pairs of parallel sides and four congruent sides; opposite angles are congruent. A rhombus is also a polygon and a parallelogram.

Right angle: an angle that measures 90 degrees

Right trapezoid: a trapezoid with two adjacent right angles. A right trapezoid is also a polygon and a quadrilateral.

Right triangle: a triangle with a right angle

Scalene triangle: a triangle with no congruent sides

Side: a line segment connected to other segments to form a polygon

Square: a quadrilateral with two pairs of parallel sides, four congruent sides and four right angles. A square is also a polygon, a parallelogram, a rectangle, and a rhombus.

Trapezoid: a quadrilateral with exactly one pair of parallel sides. A trapezoid is also a polygon.

Triangle: a three-sided polygon

Vertex: the point at which two line segments, lines, or rays meet to form an angle

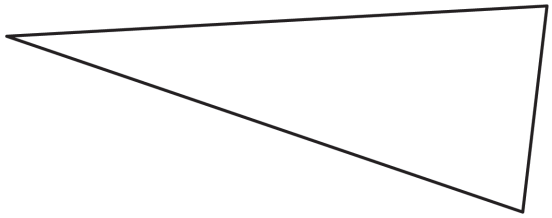
Vertices: plural of vertex

Reference: Math on Call, A Mathematics Handbook

Name _____

Summative Assessment

1. How many ways can you name or classify this shape?



_____

_____

_____

2. Draw an acute isosceles triangle.

3. What makes a right angle a right angle?

_____

_____

_____

4. What makes an obtuse angle an obtuse angle?

5. List all the properties of this shape:

6. List all the ways to name or classify the shape.

7. How do you know if two lines are perpendicular?

8. Draw an isosceles trapezoid.

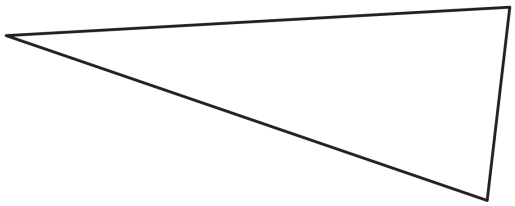
9. List the properties that make the figure that you drew an isosceles trapezoid.

10. Read the following statement: "All rhombuses are parallelograms, but only some parallelograms are rhombuses." Explain why this statement is true.

11. Read the following statement: "All squares are rectangles, but only some rectangles are squares." Explain why this statement is true.

Summative Assessment Answers

1. How many ways can you name or classify this shape?



Answer: *The shape is a polygon and an isosceles triangle.*

2. Draw an acute isosceles triangle.

Answer: *Any triangle that has three angles less than 90 degrees and two congruent sides is correct.*

3. What makes a right angle a right angle?

Answer: *A right angle is an angle that measures 90 degrees.*

4. What makes an obtuse angle an obtuse angle?

Answer: *An obtuse angle is an angle that measures more than 90 degrees and less than 180 degrees.*

5. List all the properties of this shape:

Answer: *The shape has two pairs of parallel lines, two pairs of congruent sides and four right angles.*

6. List all the ways to name or classify the shape.

   Answer: *The shape is a polygon, a quadrilateral, a parallelogram and a rectangle.*

7. How do you know if two lines are perpendicular?

   Answer: *Two lines are perpendicular if they form right angles where they meet.*

8. Draw an isosceles trapezoid.

   Answer: *Any trapezoid with two pairs of adjacent and congruent angles is correct.*

9. List the properties that make the figure that you drew an isosceles trapezoid.

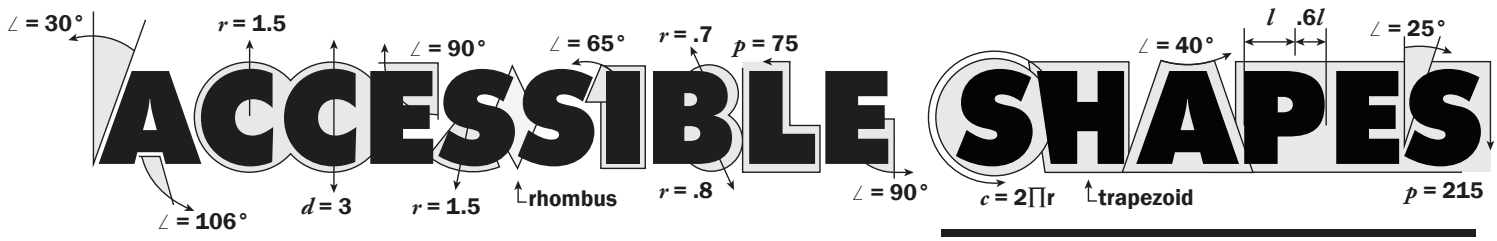
   Answer: *One pair of parallel lines and two pairs of adjacent and congruent angles.*

10. Read the following statement: "All rhombuses are parallelograms, but only some parallelograms are rhombuses." Explain why this statement is true.

    Answer: *Rhombuses and parallelograms both have two pairs of parallel sides, but rhombuses have four congruent sides and not all parallelograms have four congruent sides.*

11. Read the following statement: "All squares are rectangles, but only some rectangles are squares." Explain why this statement is true.

    Answer: *Squares and rectangles both have two pairs of parallel sides and four right angles, but squares have four congruent sides.*



GEOMETRY

Surface Area and Volume

http://www.WesternReservePublicMedia.org/geometry

Surface Area and Volume

Measurement

Grade 6

05-07 Benchmark G. *Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes.*

Y2003.CMA.S02.G05-07.BG.L06.I01 Measurement Units /

01. Understand and describe the difference between surface area and volume.

05-07 Benchmark F. *Analyze and explain what happens to area and perimeter or surface area and volume when the dimensions of an object are changed.*

Y2003.CMA.S02.G05-07.BF.L06.I01 Measurement Units /

01. Understand and describe the difference between surface area and volume.

This unit helps students to understand the difference between volume and surface area and begins with two formative assessments. Next, the unit reviews finding the area and perimeter of rectangular prisms and finding the area and circumference of cylinders. Then, students are introduced to the volume and surface area of rectangular prisms and of cylinders through the use of PowerPoint presentations and student handouts.

As a final project, the students use a sheet of 8.5" x 11" paper and make a rectangular prism by folding the paper into fourths and adding a bottom. They then turn the paper in the landscape position and do the same. They must determine which shape has a greater volume and which has a greater surface area. The next part of the lesson asks the students to repeat the process, but this time to make a cylinder using a sheet of paper, both in the portrait and the landscape position and then determine which has a greater volume. Popcorn is the tool that will help them visually see the results.

The unit is followed by a summative assessment.

Surface Area and Volume Vocabulary

Area: The size a bounded region takes up; is expressed in square units

Circumference: The distance around the edge of a circle

Diameter: The distance across a circle through its center point

Hypotenuse: The side opposite the 90-degree angle in a right triangle; also the longest side of a right triangle

Perimeter: The total distance around the outside of a polygon

Pi or π: The circumference of any circle divided by its diameter, rounded to the number 3.14

Plane: A flat surface having only two dimensions

Polygon: A closed figure formed from line segments that meet only at their endpoints.

Rectangular prism: A solid, three-dimensional object that has six faces that are rectangles

Radius: The measure from the center of a circle to a point on the circle

Slant: The diagonal distance from the top of a cone to its base

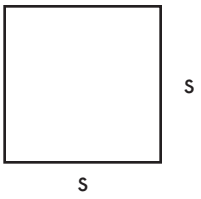
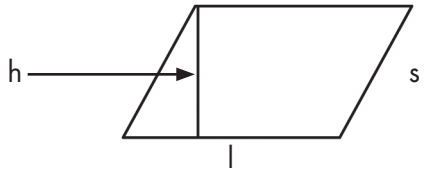
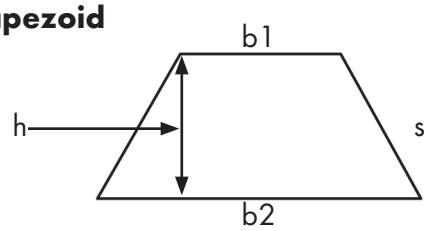
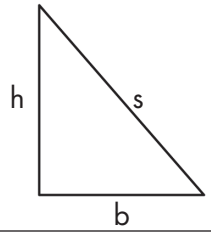
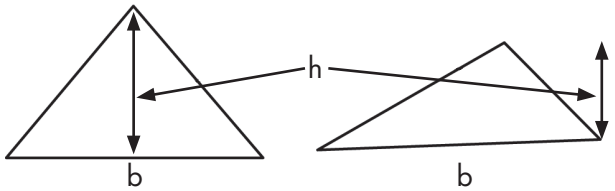
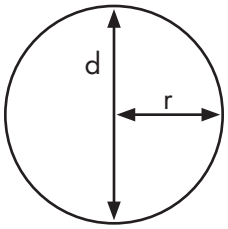
Surface area: The sum of all the areas of all surfaces of a three-dimensional object, measured in square units

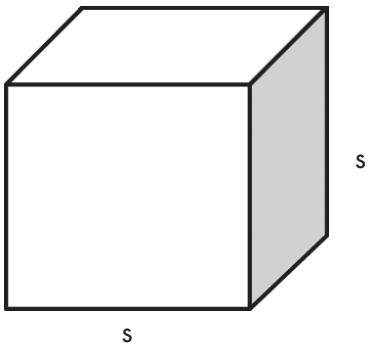
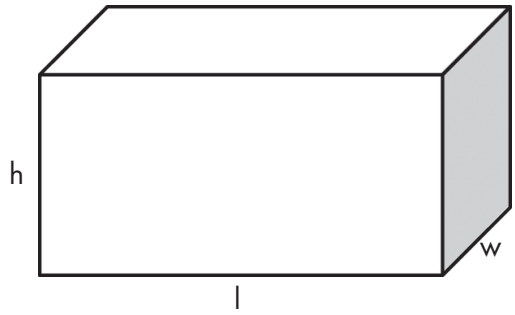
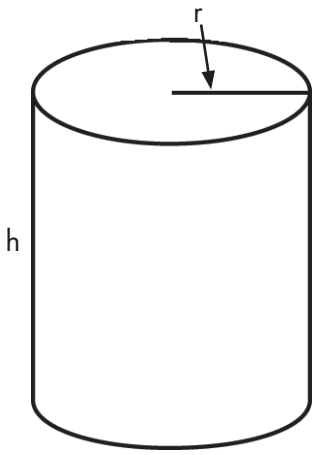
Volume: The amount of space inside a three dimensional shape, measured in cubic units

Common Abbreviations		
A = area	l = length	
b = base	P = perimeter	SA = surface area
C = circumference	π = pi	V = volume
d = diameter	r = radius	w = width
h = height	s = side	

Did you notice that the abbreviations for area (A), circumference (C), perimeter (P), surface area (SA) and volume (V) use capital letters? The remaining measures are generally lowercase letters.

Important Formulas

Two-dimensional figures or planar figures	
Square 	P = 4 · s or P = s + s + s + s A = s · s (square units)
Rectangle 	P = 2 · (l + w) or P = l + l + w + w A = l · w (square units)
Parallelogram 	P = 2 · (l + s) or P = l + l + s + s A = l · h (square units)
Trapezoid 	P = side a + side b + side c + side d A = 1/2 (b1 + b2) · h (square units)
Right Triangle 	P = side a + side b + side c A = 1/2 (b · h) (square units)
Other Triangles 	P = side a + side b + side c A = 1/2 (b · h) (square units)
Circle 	C = π · d A = π · r ^{2} (square units)

Three-dimensional figures	
Cube  	SA = 6 (4 · s) or SA = 6 · (s + s + s + s) V = s · s · s (cubic units)
Rectangular Prism  	SA = 4 (l · w) + 2 (w · h) V = l · w · h (cubic units)
Cylinder  	SA = 2($\pi \cdot r^2$) + ($\pi \cdot d \cdot h$) V = $\pi \cdot r^2 \cdot h$ (cubic units)

Review: Area, Perimeter, Circumference

Overview

Students will comprehend the concepts of area, perimeter, circumference and the use of formulas to find a solution. They will then learn the difference between surface area and volume.

Standards Addressed

Measurement

Grade 6

05-07 Benchmark

G. Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes.

Y2003.CMA.S02.G05-07.BG.L06.I05
Use Measurement Techniques and Tools /

05. Understand the difference between perimeter and area, and demonstrate that two shapes may have the same perimeter, but different areas or may have the same area, but different perimeters.

Common Core Standards Addressed

Geometry

Grade 6G

Solve real-world and mathematical problems involving area, surface area and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Part 1: Review

Materials

- *What Am I?* PowerPoint presentation
- *Formative Assessment – Vocabulary: What Am I?* student handout
- *Formative Assessment: Volume and Surface Area* student handout

Procedure

1. For a full understanding of new material, it is necessary for students to have an understanding of material presented in earlier years. This lesson begins with a formative assessment titled *What Am I?*, offered as both a PowerPoint presentation and a student handout. *Answers: 1. C, area; 2. B, circumference; 3. volume; 4. surface area; 5. diameter; 6. hypotenuse; 7. radius; 8. polygon; 9. perimeter; 10. C, pi or π; 11. volume.*

2. The students' success with *Formative Assessment: Volume and Surface Area* determineds how much of the new material they already understand.

Evaluation

This evaluation could be used in a variety of ways:

1. A percent of correct answers could be used to give a grade.

2. Students could work in teams to come up with acceptable answers.

3. The assessment could be worked through as a study sheet over everything that has been covered and then one or two problems could be given to determine understanding.

Alternative Assessment

The popcorn problems could be used as authentic assessment. If students complete the task, they can be asked to write their understanding of the process by which both activities were done. They can then be graded on the following rubric.

CATEGORY	4	3	2	1
Strategy/ Procedures	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve the problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) or is not written.
Mathematical Reasoning	Uses complex and refined mathematical reasoning.	Uses effective mathematical reasoning.	There is some evidence of mathematical reasoning.	There is little evidence of mathematical reasoning.
Use of Manipulatives	Student always listens and follows directions and only uses manipulatives as instructed.	Student typically listens and follows directions and uses manipulatives as instructed most of the time.	Student sometimes listens and follows directions and uses manipulatives appropriately when reminded.	Student rarely listens and often "plays" with the manipulatives instead of using them as instructed.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.

Part 2: Area and Perimeter of Polygons

Materials

- *Area and Perimeter* PowerPoint presentation
- *Area and Perimeter* student handout
- Calculators

Procedure

1. Review the formative assessment and determine if students need to spend more time on the key concepts of area, perimeter, circumference, volume and surface area. If so, use the *Area and Perimeter* presentation or handout.

Answers

What did the duck say to the store clerk when buying chapstick?

J	U	S	T		P	U	T		I	T		O	N		M	Y		B	I	L	L	!
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

Perimeters		Area	
34 cm	20-I	100 m^2	21-L
31 m	14-N	60 cm^2 or m^2	1-J
40 ft	16-M	60 m^2 or cm^2	7-U
24 m	6-P	36 ft^2	8-T
11 cm	19-B	56 in^2	3-S
16 ft	10-I	42 m	22-L
50 m	17-Y	6 in^2	23-!
12 in	11-T	96 ft^2	4-T
33 ft	5-Z	37 cm^2	12-R
32 in	13-O	3 cm^2	2-U
14 m	9-K	20 ft^2	15-M

Part 3: The Concept of Pi (π)

Materials

- Many circular items (cans, jars, lids, etc.) in a variety of sizes
- Tape measures
- Calculators
- *Pi* student handout

Procedure

1. Display a large variety of circles, tape measures and calculators. The circles should be numbered in some way so students can compare their answers.

2. Allow the students to work with a partner. Each team will collect some circles, a tape measure, a calculator and a *Pi* student handout.

3. Instruct students to measure the diameter and circumference and then divide them and record the information on the student handout.

4. After they are finished, the students can discuss the results as a class. They should have found that the circumference is about three times bigger than the diameter and that if divided on the calculator, the result is 3.14. There will be some variety in the answers. Ask why. Students should be able to say it is because of the accuracy of their measurements.

5. Ask if some students measured in centimeters and others in inches. If so, was there a difference in the result? *No, it will always measure 3.14 regardless of the unit of measure.*

Part 4: Area and Circumference of Circles

Materials

- *Circles: Area and Circumference* PowerPoint presentation
- *Review: Circles* student handout
- Calculators

Procedure

1. Now that the students have reviewed the concept of pi, review how pi is used to find the area and the circumference of a circle.

2. Review what diameter and radius are and when you use each one.

3. Review the formulas for area and circumference of circles.

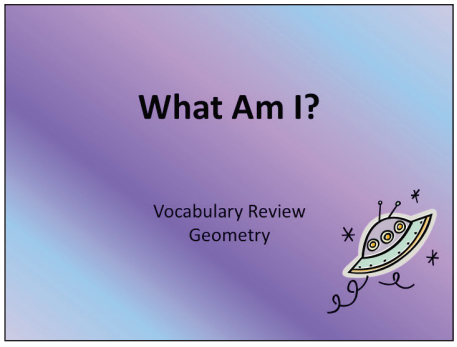
4. Allow the students to work with a partner to complete the *Review: Circles* student handout.

Answers

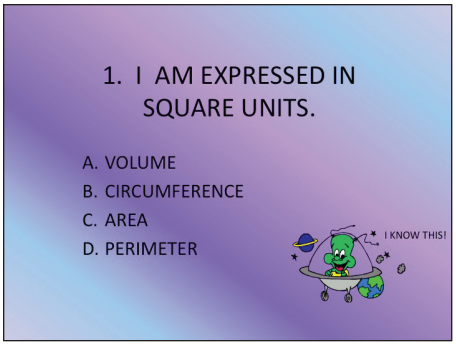
1. 2.5 in
2. 15 in
3. 4.5 ft
4. 28.26 m^2
5. 18.84 m
6. 50.52 ft^2
7. 25.12 ft
8. 78.5 in^2
9. 31.4 in
10. 5,024 m^2
11. 251.2m
12. 452.16 fr^2
13. 75.36 ft
14. 63.585 in^2
15. 28.26 in

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
W	I	T	H		H	A	R	E		S	P	R	A	Y

What Am I?
PowerPoint Presentation



Slide 1



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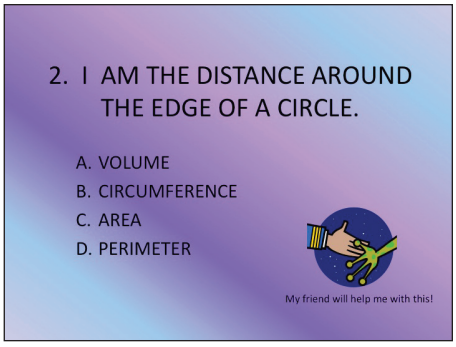

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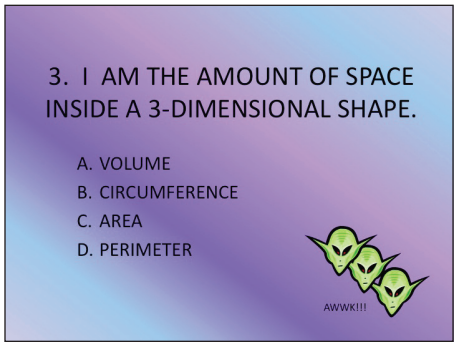

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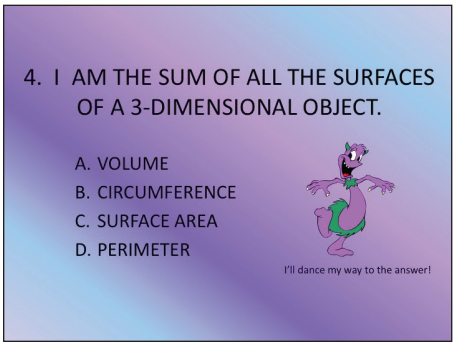

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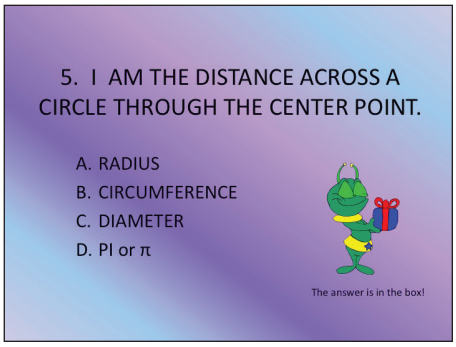

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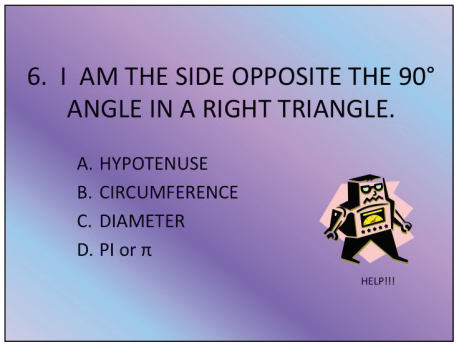

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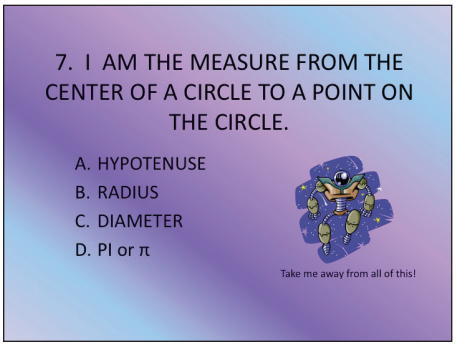

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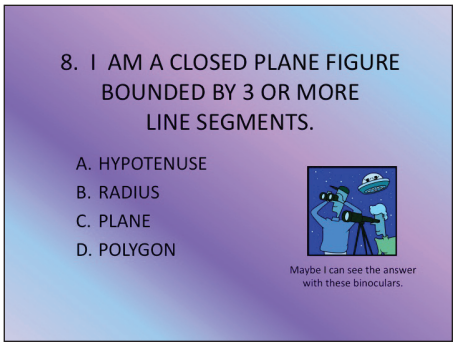
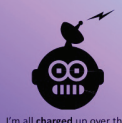


Slide 9

9. I AM THE DISTANCE AROUND THE OUTSIDE OF A POLYGON.

A. HYPOTENUSE
B. RADIUS
C. PLANE
D. PERIMETER



Slide 10

10. I HAVE A VALUE OF 3.14

A. HYPOTENUSE
B. RADIUS
C. PI or π
D. PERIMETER

Slide 11

11. I AM ALWAYS MEASURED IN CUBIC UNITS.

A. HYPOTENUSE
B. VOLUME
C. AREA
D. PERIMETER

Slide 12

ANSWERS

Slide 13

1. I AM EXPRESSED IN SQUARE UNITS.

C. AREA

Slide 14

2. I AM THE DISTANCE AROUND THE EDGE OF A CIRCLE.

B. CIRCUMFERENCE

Slide 15

3. I AM THE AMOUNT OF SPACE INSIDE A 3-DIMENSIONAL SHAPE.

A. VOLUME

Slide 16

4. I AM THE SUM OF ALL THE SURFACES OF A 3-DIMENSIONAL OBJECT.

C. SURFACE AREA

Slide 17

5. I AM THE DISTANCE ACROSS A CIRCLE THROUGH THE CENTER POINT.

C. DIAMETER

Slide 18

6. I AM THE SIDE OPPOSITE THE 90° ANGLE IN A RIGHT TRIANGLE.

A. HYPOTENUSE

Slide 19

7. I AM THE MEASURE FROM THE CENTER OF A CIRCLE TO A POINT ON THE CIRCLE.

B. RADIUS

Slide 20

8. I AM A CLOSED PLANE FIGURE BOUNDED BY 3 OR MORE LINE SEGMENTS.

D. POLYGON

Slide 21

9. I AM THE DISTANCE AROUND THE OUTSIDE OF A POLYGON.

D. PERIMETER

Slide 22

10. I HAVE A VALUE OF 3.14

C. PI or π

Slide 23

11. I AM ALWAYS MEASURED IN CUBIC UNITS.

B. VOLUME

Slide 24

Slide 25

Name _____

Formative Assessment — Vocabulary

What Am I?

Please circle the correct answer.

1. I am expressed in square units.
    a. volume
    b. circumference
    c. area
    d. perimeter

2. I am the distance around the edge of a circle.
    a. volume
    b. circumference
    c. area
    d. perimeter

3. I am the amount of space inside a three-dimensional shape.
    a. volume
    b. circumference
    c. area
    d. perimeter

4. I am the sum of all the surfaces of a three-dimensional object.
    a. volume
    b. circumference
    c. surface area
    d. perimeter

5. I am the distance across a circle through the center point.
    a. radius
    b. circumference
    c. diameter
    d. pi or π

6. I am the side opposite the 90 degrees angle in a right triangle.
    a. hypotenuse
    b. circumference
    c. diameter
    d. pi or π

7. I am the measure from the center of a circle to a point on the circle.
    a. hypotenuse
    b. radius
    c. diameter
    d. pi or π

8. I am a closed-plane figure bounded by three or more line segments.
    a. hypotenuse
    b. radios
    c. plane
    d. polygon

9. I am the distance around the outside of a polygon.
    a. hypotenuse
    b. radius
    c. plane
    d. perimeter

10. I have the value of 3.14
    a. hypotenuse
    b. radius
    c. pi or π
    d. perimeter

11. I am always measured in cubic units
    a. hypotenuse
    b. volume
    c. area
    d. perimeter

Formative Assessment

Volume and Surface Area

1. What is volume?

2. What is surface area?

3. Write two facts about rectangular prisms.

   a.

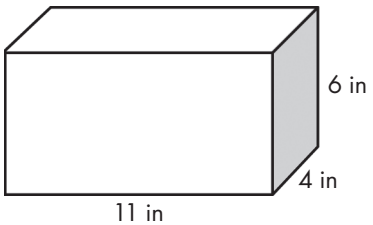
   b.

4. Write two facts about cylinders.

   a.

   b.

5. Given this rectangular prism:



   a. Find the volume. Show all work.

   b. Find the surface area. Show all work.

6. What is the value of pi (π)? _____ How can you find pi (π)?

7. What is the diameter of a circle?

8. What is the radius of a circle?

9. What is the circumference of the circle?

10. The measure of a volume is always **squared** or **cubed** (select one).

11. The measure of surface area is always **squared** or **cubed** (select one).

BONUS:

Write the formula for the volume of a rectangular prism.

Write the formula for the surface area of a rectangular prism.

Formative Assessment — Answers

Volume and Surface Area

1. What is volume?

   Amount inside a shape

2. What is surface area?

   Amount it takes to cover a shape

3. Choose two features that describe rectangular prisms.

   a. has six sides

   b. opposite sides are equal

   c. angles are 90 degrees

   d. opposite sides are parallel

   e. has length, width and height

4. Choose two features that describe cylinders.

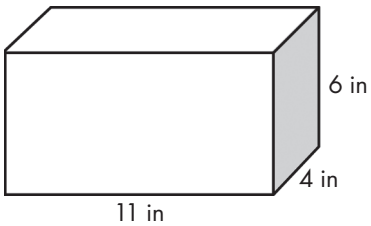
   a. looks like a can

   b. has two circles (top and bottom)

   c. circles are equal size

   d. has a cover or a height

5. Given this rectangular prism:



   a. Find the volume. Show all work.

   $V = l \cdot w \cdot h$

   $11 \times 6 \times 4 = 264\ in^3$ *or 264 cubic inches*

   b. Find the surface area. Show all work.

   $SA = 2lw + 2lw + 2lw$

   $(2 \times 11 \times 4) + (2 \times 4 \times 6) + (2 \times 11 \times 6)$

   $88 + 48 + 132$

   $SA = 268\ in^2$ *or 268 square inches*

6. What is the value of pi (π)? How can you find pi (π)?

   Pi = 3.14. Divide the circumference by the diameter.

7. What is the diameter of a circle?

   Line that goes from side to side of a circle through the center — twice the radius

8. What is the radius of a circle?

   Distance from the midpoint to the edge of circle — half the diameter

9. What's the circumference of the circle?

   Distance around the outside of a circle

10. The measure of a volume is always **squared** or **cubed** (select one).

    cubed

11. The measure of surface area is always **squared** or **cubed** (select one).

    squared

BONUS: *There are variations of these formulas. As long as they express the same result, the answers are acceptable.*

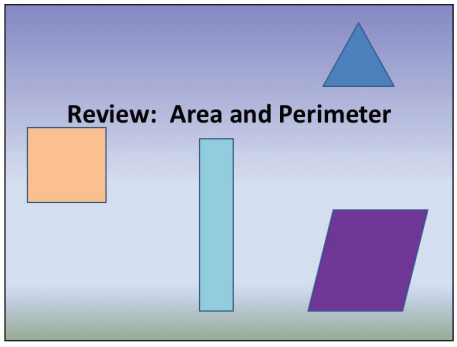
Write the formula for the volume of a rectangular prism.

$V = l \cdot w \cdot h$

Write the formula for the surface area of a rectangular prism.

$SA = 2lw + 2lw + 2lw$

Review: Area and Perimeter PowerPoint Presentation



Slide 1

Definitions

1. What is a polygon?
2. What does perimeter mean?
3. What does area mean?

Slide 2

Definitions

1. What is a polygon?
   A closed shape made up of line segments
2. What does perimeter mean?
   Distance around the outside of a polygon
3. What does area mean?
   Squares it takes to cover a shape

GIVE YOURSELF 1 POINT FOR EACH CORRECT ANSWER.

Slide 3

Finding Area and Perimeter

4. What's one way to find the perimeter of a polygon?
5. What's one way to find the area of a rectangle or square?
6. What's another way to write square inches?

Slide 4

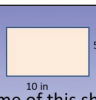
Finding Area and Perimeter

4. What's one way to find the perimeter of a polygon?
   Add the sides together.
5. What's one way to find the area of a rectangle or square?
   Multiply the length times the width
6. What's another way to write square inches?
   in^2 (use the exponent 2)

GIVE YOURSELF 1 POINT FOR EACH CORRECT ANSWER.

Slide 5

Review



7. What is the name of this shape?
8. What is the perimeter?
9. What is the area?

Slide 6

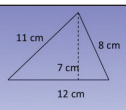
Review

7. What is the name of this shape?
   The shape is a RECTANGLE.
8. What is the perimeter?
   The PERIMETER is the distance around the edge. So add 5 + 5 + 10 + 10 and get **30 inches**.
9. What is the area?
   The area is the # of squares that cover the shape. So, multiply 10 x 5 and you get **50 square inches or $50in^2$.**

GIVE YOURSELF 1 POINT FOR EACH CORRECT ANSWER.

Slide 7

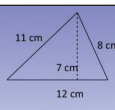
Review



10. What is the name of this shape?
11. What is the broken line called?
12. How do you find the area?

Slide 8

Review

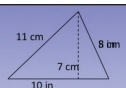


10. What is the name of this shape?
    A triangle
11. What is the broken line called?
    The altitude or the height
12. How do you find the area?
    Multiply ½ times the base times the height or $A = ½ (b \cdot h)$

GIVE YOURSELF 1 POINT FOR EACH CORRECT ANSWER.

Slide 9

Review

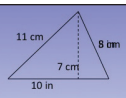


13. What is the perimeter?

14. What is the area?

Slide 10

Review

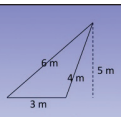


13. What is the perimeter?
    P = 11 + 8 + 7 or **26 cm**
14. What is the area?
    A = ½ (b · h) or ½ (8 x 10) or ½ (80) or
    40 cm² or 40 sq cm

GIVE YOURSELF 1 POINT FOR EACH CORRECT ANSWER.

Slide 11

Review

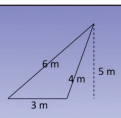


15. What is the perimeter?

16. What is the area?

Slide 12

Review

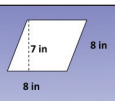


15. What is the perimeter?
    P = 6 + 4 + 3 or 13 m
16. What is the area?
    A = ½ (b · h) or ½ (6 x 5) or ½ (30) or
    15 m² or 15 sq m

GIVE YOURSELF 1 POINT FOR EACH CORRECT ANSWER.

Slide 13

Review



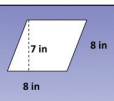
17. What is the name of this shape?

18. What is the perimeter?

19. What is the area?

Slide 14

Review



17. What is the name of this shape?
    A rhombus or a parallelogram
18. What is the perimeter?
    P = 8 + 8 + 8 + 8 or 4 (8) or 32 in
19. How do you find the area?
    Multiply the base times the height
    or A = b · h or A = 7 x 8 or 56 sq in or 56 in²

GIVE YOURSELF 1 POINT FOR EACH CORRECT ANSWER.

Slide 15

How did you do?

- A = 19-17
- B = 16-15
- C = 14-13
- D = 12-11
- This was just practice. Now you know what to study!
- Which sound matches your score?

—Sound 1 —Sound 2

Slide 16

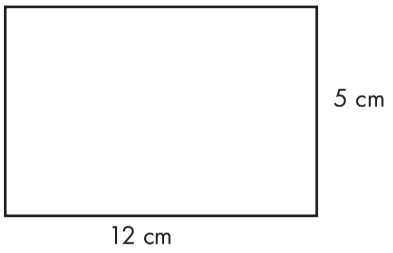
Name _____

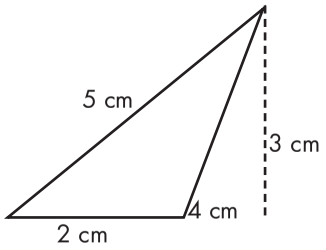
Review: Area and Perimeter

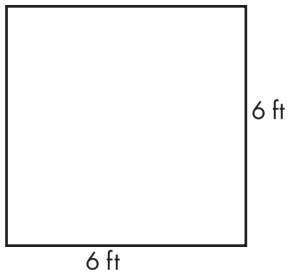
Find both the area and the perimeter of each figure. Find the answer in the appropriate answer column. Fill in the correct unit of measure for each answer, and write the letter in the matching numbered box at the bottom of the page.

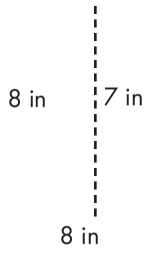
What did the duck say to the store clerk when buying chapstick?

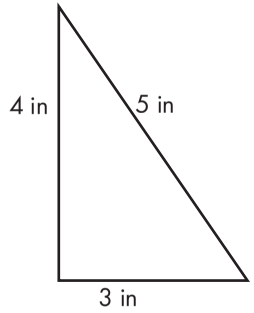
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23





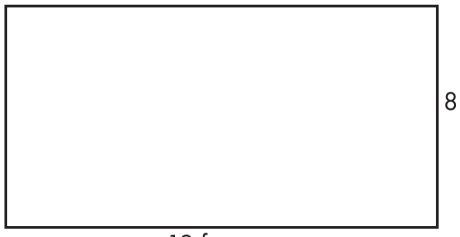


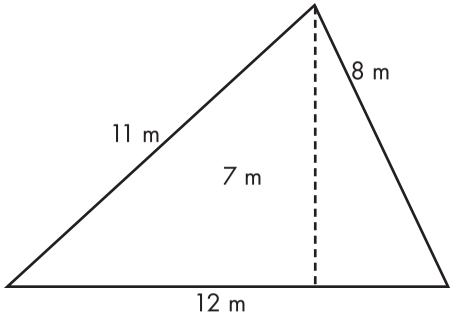


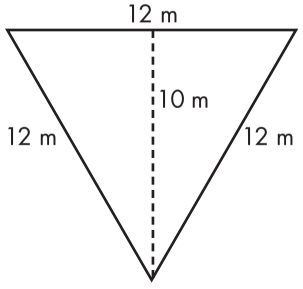


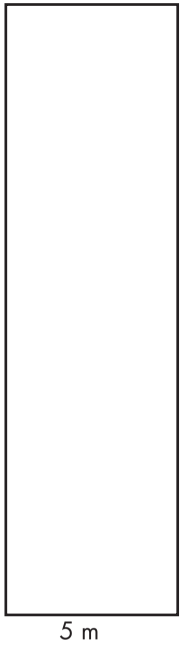
Perimeters		Area	
34	20-I	100	21-L
31	14-N	60	1-J
40	16-M	60	7-U
24	6-P	36	8-T
11	19-B	56	3-S
16	10-I	42	22-L
50	17-Y	6	23-!
12	11-T	96	4-T
33	5-Z	37	12-R
32	13-O	3	2-U
14	9-K	20	15-M

You won't use every answer above.









Name _____

Pi — π

Your Job:

1. Collect one circle, a tape measure and a calculator.
2. Write the number of the circle you collected.
3. Measure the circumference (distance around the edge) of the circle and record your result.
4. Measure the diameter (distance across the circle from side to side through the midpoint) and record your answer.
5. Using your calculator, divide the circumference by the diameter and record your answer.

$$\frac{\textbf{Circumference}}{\textbf{Diameter}}$$

Number Selected	Circumference	Diameter	Result

6. Write two sentences about what you have found.

Circles: Area and Circumference PowerPoint Presentation

Circles: Area and Circumference

Slide 1

Definitions

- **Circumference:** Distance around the outside of a circle

- **Area:** How many squares it takes to cover a circle

Slide 2

Definitions

- **Radius:** A line segment that goes from the center of the circle to the edge of the circle.

radius   **diameter**

- **Diameter:** Goes from edge to edge through the middle.
  - **Diameter** = 2 times radius or **d = 2r**
  - **Radius** = ½ times diameter or **r = d/2**

Slide 3

Definitions

- **Pi:** Measure of the distance around the circle / (divided by) the diameter or distance across the circle and you **ALWAYS** get 3.14159265358979323846...
- We round this to **3.14** and call it **Pi** or use the symbol for pi which is **π**.

Slide 4

Definitions

Circumference: If you know the diameter of a circle you can find the circumference by using this formula:

$C = pi \cdot d$ or $C = \pi d$

But what if you only know the radius?

$C = pi \cdot 2 \cdot r$ or $C = 2\pi r$

Slide 5

Definitions

Area: The number of squares it takes to cover a circle

$A = pi \cdot r \cdot r$ or $A = \pi r^2$

But what if you only know the radius?

Divide the diameter in half to get the radius and then use the formula.

Slide 6

Examples

- Example 1: The radius of a circle is 2 inches. What is the diameter?
  - d = 2 · r
  - d = 2 · (2 in)
  - d = 4 in

Slide 7

Examples

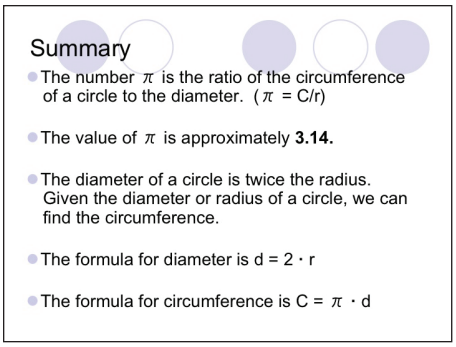
- Example 2: The diameter of a circle is 3 centimeters. What is the circumference?
  - C = π · d
  - C = 3.14 · 3 cm
  - C = 9.42 cm

Slide 8

Examples

- Example 3: The radius of a circle is 2 inches. What is the circumference?
  - d = 2 · r
  - d = 2 · (2 in)
  - d = 4 in
  - C = π · d
  - C = 3.14 · (4 in)
  - C = 12.56 in

Slide 9


Summary
The number π is the ratio of the circumference of a circle to the diameter. (π = C/r)
The value of π is approximately 3.14.
The diameter of a circle is twice the radius. Given the diameter or radius of a circle, we can find the circumference.
The formula for diameter is d = 2 · r
The formula for circumference is C = π · d

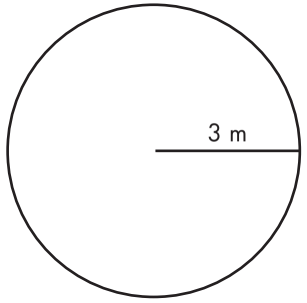
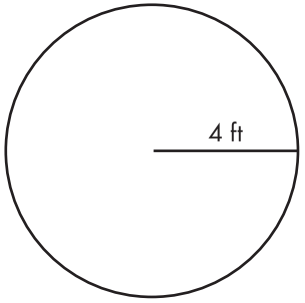
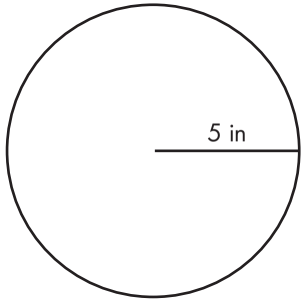
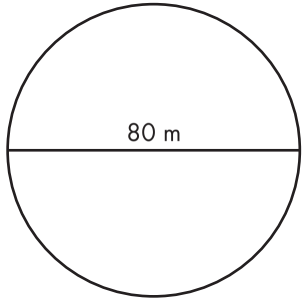
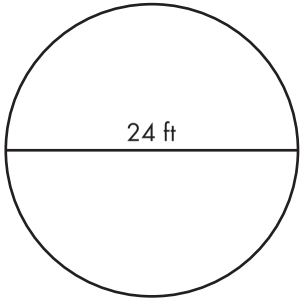
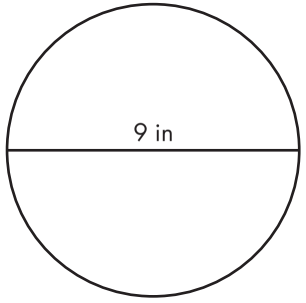
Slide 10

Name _____

Review: Circles

Find what is asked for in each problem. Use 3.14 for π. Find your answer in the answer columns. Write the letter of the answer in the space containing the number of the exercise. Remember **$A = \pi r^2$ and $C = \pi d$.**

What is the best way to paint a rabbit?

1. d = 5 in; r = _____in	2. r = 7.5 in; d = _____in	3. d = 9 ft; r = _____ft
		
4. A = _____ 5. C = _____	6. A = _____ 7. C = _____	8. A = _____ 9. C = _____
		
10. A = _____ 11. C = _____	12. A = _____ 13. P = _____	14. A = _____ 15. C = _____

Answer Column	
	5,024 m^2
H	50.52 ft^2
Y	28.26 in
W	2.5 in
R	75.36 ft^2
A	25.12 ft
A	63.585 in^2
I	15 in
P	452.16 ft^2
R	78.5 in^2
S	251.2 m
T	4.5 ft
	18.84 m
E	31.4 in
H	28.26 m^2

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Graphing Area and Perimeter

Overview

Students will work in groups and build every possible rectangular shape with blocks and record both the area and the perimeter. They will then graph the results and make a generalization about their findings.

Standards Addressed

Measurement

Grade 6

05-07 Benchmark

G. Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes.

Y2003.CMA.S02.G05-07.BG.L06.I05 Use Measurement Techniques and Tools /

05. Understand the difference between perimeter and area, and demonstrate that two shapes may have the same perimeter, but different areas or may have the same area, but different perimeters.

Materials

- 24 blocks per group
- *Graphing Area and Perimeter – Fixed Perimeter* student handout
- *Graphing Area and Perimeter – Fixed Area* student handout
- Graph paper
- Multicolored markers

Procedure

Fixed Perimeter

1. Divid the class into groups of two or three. Each group should get 24 blocks and copies of the *Graphing Area and Perimeter – Fixed Perimeter* student handout. All group members should have their own handout even though they are working together.

2. Their task is to follow this scenario:

   Your dad wants to build a bus shelter for you so that you don't have to stand in the mud and snow. He has enough wood to build a rectangular shelter whose floor has a total perimeter of 20 feet.

Students are to fill in the table of all the whole number possibilities for the length and width of the shelter. REMEMBER: Perimeter is the distance around the outside edge, so p= 2L + 2W and area is how many squares cover the area: A = l · w.

3. Students should arrange the blocks to form as many rectangles with a perimeter of 20 as they can. They do not have to use all of the blocks. Each time that the blocks have been arranged, the students should draw the rectangle on graph paper and label the measurements. This is helpful for students to see all of the rectangles that they have created. Students can always use practice at labeling their work. Drawing also helps them to better understand the difference between area and perimeter. Sixth graders still struggle with the difference between linear measurement and measuring in two dimensions.

4. The students should then create a graph using the data that they collected. The graph will look like an upside-down "U" (see answer sheet).

5. Ask the students to make some generalizations about the data they collected. They could say that the length and width together equal 10. The goal is for them to figure out that the squarer the shape, the greater the area.

Procedure

Fixed Area

1. Divid the class into groups of two or three. Ask each group to send someone to get 12 blocks and the *Graphing Area and Perimeter – Fixed Area* student handout. Everyone on the team should have their own handout even though they are working together.

2. There task is to follow this scenario:

   Your dad wants to construct a fence for the family dog. He wants to buy as little fencing as possible, so he wants to find the smallest perimeter for the rectangular space. Each piece of fencing equals one square foot. Using 12 blocks, with each block representing one square foot of fencing construct a place for the dog to be outside.

   Students are to fill in the table of all the whole number possibilities for the length and width of the shelter. REMEMBER: Perimeter is the distance around the outside edge so p= 2L + 2 W and area is how many squares cover the area. A = l · w.

3. Each time the tiles have been arranged the students should draw the rectangle on graph paper and label the measurements. This is helpful for students to see all of the rectangles that they have created. Students can always use practice at labeling their work. Drawing also helps students to see the difference between area and perimeter more clearly. Sixth graders still struggle with the difference between linear measurement and measuring in two dimensions.

4. Students should then create a graph using the data that they collected. The graph will look like a "U" (see answer sheet).

5. Ask the students to make some generalizations about the data they collected. Write the generalizations where they can see them. *Answers may be that there are two of every perimeter – 26, 16 and 14. It is hoped that someone will make the generalization that the closer the shape is to a square, the smaller the perimeter.*

Evaluation

Ask the following question: If each child requires four square feet (or any amount that you choose) of space in the bus shelter, what is the highest number of children a shelter will hold? This question can be asked of any of the various sizes of shelters. If the rectangles are already drawn on graph paper, students can easily see how many children will fit in each shelter (four square feet is four squares). This question also serves to make the activity less abstract for students.

Students can either write their response to these questions or they can explain the answers to you or to the class. The rubric below can be used to evaluate the explanations.

CATEGORY	4	3	2	1
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s), or is not written.
Mathematical Reasoning	Uses complex and refined mathematical reasoning.	Uses effective mathematical reasoning.	Some evidence of mathematical reasoning.	Little evidence of mathematical reasoning.
Explanation	Explanation is detailed and clear.	Explanation is clear.	Explanation is a little difficult to understand, but includes critical components.	Explanation is difficult to understand and is missing several components, or was not included.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.

Name _____

Graphing Area and Perimeter — Fixed Perimeter

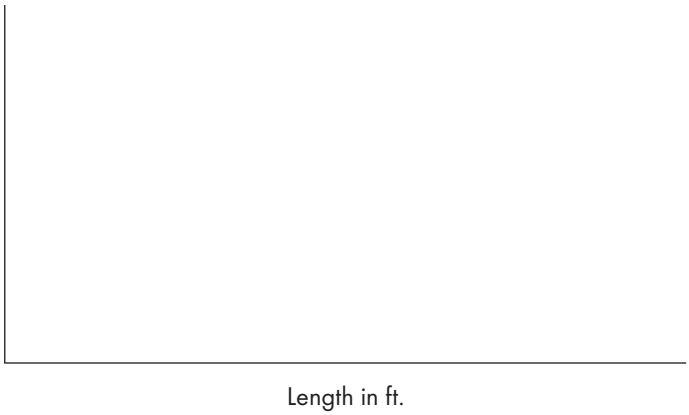
Problem: Your dad wants to build a bus shelter for you so that you don't have to stand in the mud and snow. He has enough wood to build a rectangular shelter whose floor has a total perimeter of 20 feet.

1. Make a table of all the whole number possibilities for the length and width of the shelter. REMEMBER: Perimeter is the distance around the outside edge, so p= 2L + 2 W and area is how many squares cover the area. A = l · w.

Length	Width	Perimeter	Area
Example: 1	9	20	9

2. What dimension should your dad choose to make the bus shelter?

3. Graph the length on the x-axis and the area on the y-axis from your table on the graph provided.



4. Is there any generalization you can make about which dimension you should select?

Graphing Area and Perimeter — Fixed Perimeter Answers

Problem: Your dad wants to build a bus shelter for you so that you don't have to stand in the mud and snow. He has enough wood to build a rectangular shelter whose floor has a total perimeter of 20 feet.

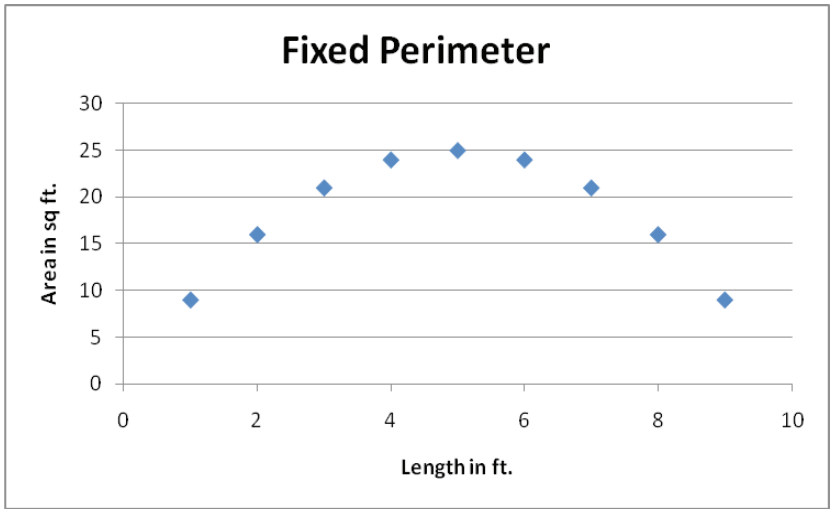
1. Make a table of all the whole number possibilities for the length and width of the shelter. REMEMBER: Perimeter is the distance around the outside edge, so p= 2L + 2 W and area is how many squares cover the area. A = l · w.

Length	Width	Perimeter	Area
Example: 1	9	20	9
2	8	20	16
3	7	20	21
4	6	20	24
5	5	20	25
6	4	20	24
7	3	20	21
8	2	20	16
9	1	20	9

2. What dimension should your dad choose to make the bus shelter?

   5 x 5 has the biggest area, so he should choose this one.

3. Graph the length on the x-axis and the area on the y-axis from your table on the graph provided.



4. Is there any generalization you can make about which dimension you should select?

   The more square it is, the greater the area.

Name _____

Graphing Area and Perimeter — Fixed Area

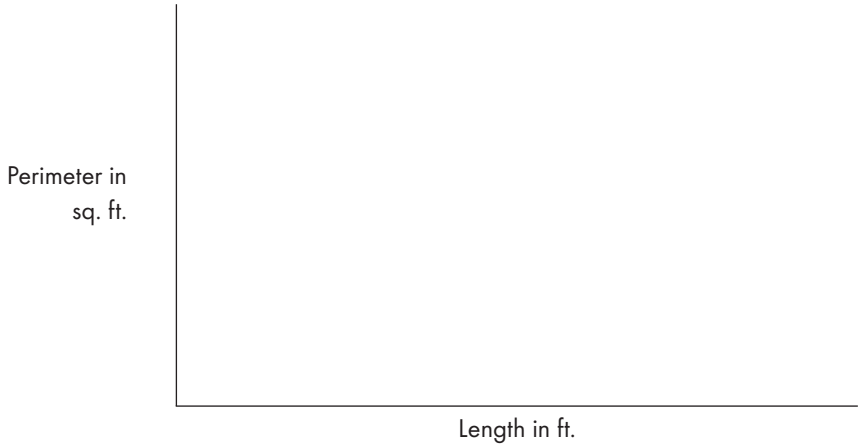
Problem: Your dad wants to construct a fence for the family dog. He wants to buy as little fencing as possible, so he needs to find the smallest perimeter for the rectangular space. Each piece of fencing equals one square foot. Using 12 blocks, with each block representing one square foot of fencing construct a place for the dog to be outside.

1. Make a table of all the whole number possibilities for the length and width of the outdoor rectangular space for the dog. REMEMBER: Perimeter is the distance around the outside edge, so p= 2L + 2 W and area is how many squares cover the area. A = l · w.

Length	Width	Perimeter	Area
Example: 1	12	26	12

2. What dimension should your dad choose if he wants to spend as little as possible on fencing?

3. Graph the length on the x-axis and the area on the y-axis from your table on the graph provided.



4. Is there any generalization you can make about which dimension you should select?

Graphing Area and Perimeter — Fixed Area Answers

Problem: Your dad wants to construct a fence for the family dog. He wants to buy as little fencing as possible, so he needs to find the smallest perimeter for the rectangular space. Each piece of fencing equals one square foot. Using 12 blocks, with each block representing one square foot of fencing construct a place for the dog to be outside.

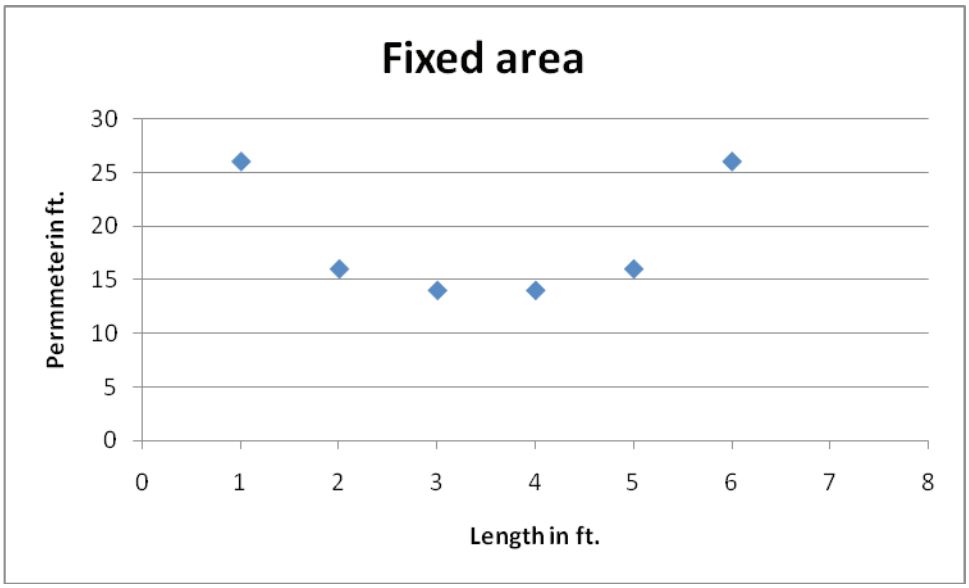
1. Make a table of all the whole number possibilities for the length and width of the outdoor rectangular space for the dog. REMEMBER: Perimeter is the distance around the outside edge, so p= 2L + 2 W and area is how many squares cover the area. A = l · w.

Length	Width	Perimeter	Area
Example: 1	12	26	12
2	6	16	12
3	4	14	12
4	3	14	12
6	2	16	12
12	1	26	12

2. What dimension should your dad choose if he wants to spend as little as possible on fencing?

   Either 4 x 3 or 3 x 4. These have the smallest perimeter.

3. Graph the length on the x-axis and the area on the y-axis from your table on the graph provided.



4. Is there any generalization you can make about which dimension you should select?

   The closest to a square gives you the smallest perimeter.

Introduction to Volume and Surface Area

Overview

This lesson provides an introduction to finding the volume and surface area of rectangular prisms and cylinders. Students conduct a hands-on activity where they make models of both the volume and surface area of rectangular prisms and cylinders. They then complete student handouts for each concept.

Standards Addressed

Measurement

Grade 6

05-07 Benchmark

G. *Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes.*

Y2003.CMA.S02.G05-07.BG.L06.I05 Use Measurement Techniques and Tools /

05. Understand the difference between perimeter and area, and demonstrate that two shapes may have the same perimeter, but different areas or may have the same area, but different perimeters.

Part 1: Volume of Rectangular Prisms

Materials

- Blocks
- Calculators
- *Volume of a Rectangular Prism* PowerPoint presentation
- *Volume of a Rectangular Prism* student handout

Procedure

1. Introduce how to find the volume of rectangular prisms.
2. View the *Volume of a Rectangular Prism* PowerPoint presentation.
3. Break the students into groups. Give each group eight blocks and tell them to make a rectangular prism with them. Review that a prism should look like a box. (It doesn't have bends.) Ask the students to display their prisms and show that there could be several types of prisms that could be made (8 x 1; 4 x 2 and 2 x 4).

4. Ask the students if they know what volume is. *(Volume is the amount of space (cubes) inside a three-dimensional figure.)* In each case, what was the volume of the shape? *(The volume was always eight regardless of the arrangement of the blocks because there were eight cubes in the prism that they made.)*

5. Ask the students to come and get some more blocks (their choice of how many) and to build a rectangular prism with them and write the volume down. *(There will be a variety of prisms made.)* Have them draw their prisms on the board or on a large paper.

6. Ask how they knew what the volume was *(how many blocks they used)* and if there could be a way they could find the volume without counting the blocks. Given enough examples, they will come up with the formula $V = l \times w \times h$.

7. Ask how they will label the volume. *(If they don't use the word "cube," ask what they used when they made their building. They counted the "cubes;" therefore, their answer should show that it is a cubic measure.)*

8. Pass out the *Volume of a Rectangular Prism* student handout. Allow them to work with a partner to finish the sheet.

Answers for Volume of a Rectangular Prism handout

1. 216 m^3
2. 600 ft^3
3. 270 m^3
4. 3600 cm^3
5. 2640 ft^3
6. 9,750,000 ft^3
7. 5 ft^3
8. False
9. 6

	O	O	O	M		P	A	P	A				
5 ft^2	5 ft^3	False	2640	600	True	216	270	3600	9,750,000	5000	7	5 ft	275

Part 2: Surface Area of Rectangular Prisms

Materials

- Boxes in a variety of sizes and shapes
- Paper
- Tape
- Tape measures
- Calculators
- *Surface Area of a Rectangular Prism* student handout

Procedure

1. Allow the students to work with a partner or a group of three.
2. Have the students collect a box, tape and enough paper to cover their box. Tell them their job is to cover the box.
3. After this is done, while the boxes are covered, ask if they can tell you how much paper it took to cover one side of the box. Because it is covering a rectangle, this is the area.
4. Ask how many sides there are on the box *(six)*.
5. Ask how much paper it would take to cover the surface of the entire box. Ask what this measure might be called *(surface area)*.
6. Ask them to find the surface area of their box. They should already know that to find the area of a rectangle you would take the length times the width using the formula $A = l \cdot w$. They would need to add together the areas of all six sides to find the surface area. Ask what the formula might look like. Have them write what they think so that the whole class can see it. Some possibilities include the following:
    a. $SA = l \cdot w + l \cdot w + l \cdot w + l \cdot w + l \cdot w + l \cdot w$
    b. $SA = 2(l \cdot w) + 2(l \cdot w) + 2 (l \cdot w)$
7. Ask how the answer would be labeled *(square measure)*.
8. Distribute the *Surface Area of a Rectangular Prism* student handout. Students can work alone or with a partner to complete this handout. Review the answers.
9. A PowerPoint presentation, *Surface Area of Rectangular Prisms*, is provided if the students need help to understand.

Answers

1. 236 m^2
2. 188 ft^2
3. 164 m^2
4. 592 cm^2
5. 1,160 ft^2
6. 164 m^2
7. 5 ft^2
8. True
9. 6
10. Free

	T	H	E		E	L	F	-	A	B	E	T
5 ft^3	5 ft^2	216	592	False	236	1160	188	5 ft	164	Free Space	True	6

Part 3: Volume of a Cylinder — Enrichment

Materials

- Cans that are small enough to be covered with two sheets of paper
- Calculators
- Paper to cover cans
- Tape
- Tape measures
- *Volume of a Cylinder* student handout

Procedure

1. Allow the students to work with a partner or in groups of three.

2. Have the students collect a can, tape and two sheets of paper. Tell them their job is to cover the can.

3. After this is done, while the cans are covered, ask if they can tell you what the amount of soup in the can is called *(volume)*. The volume therefore must have something to do with how tall the can is (height) and how wide the can is. Hopefully someone will say that the width of the can is the diameter of the circle. So, to find the volume of the can, they would have to find the area of the circle times the height.

4. Ask them to find the volume of their can. First, they must measure the diameter of the can and then from that find the radius. The formula for finding the area of a circle is pi times the radius squared. The formula is $V = \pi r^2$.

5. To find the volume they would take the area of the circle and multiply it times the height. The formula would be $V = \pi r^2 \cdot h$.

6. Distribute the *Volume of a Cylinder* student handout. Students can work alone or with a partner to complete this handout. Go over the answer.

Answers for Volume of Cylinders

1. 169.56 in^3
2. 150.72 cm^3
3. 62.8in^3
4. 942 cm^3
5. 8.4 gal
6. 863.5 cm^3
7. 10 ft^3
8. True
9. 3815.1 cm^3

	H	I	S	S	-	T	O	R	Y		
10 ft	150.72	True	62.8	863.5	10 ft^3	169.56	942	3815.1	8.4	10 ft^2	False

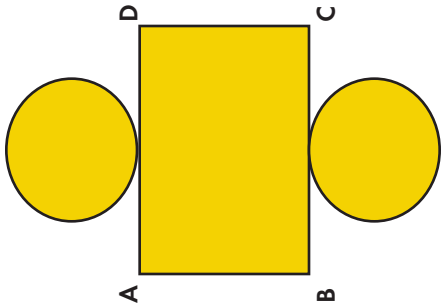
Part 4: Surface Area of a Cylinder — Enrichment

Materials

- Cans that the students covered in the volume activity
- Tape measure
- *Surface Area of a Cylinder* student handout
- *Surface Area of a Cylinder* PowerPoint presentation

Procedure

1. Ask the students to get the can that they covered when they found the volume and carefully remove the paper so they get two circles and a rectangle. Their pieces should look like this:



2. Review radius and diameter. Ask what they think finding the surface area of the volume is. Review finding the area and circumference of a circle. Review finding the area of a rectangle.

3. Ask the students if the length of the rectangle is the same as the circumference of a circle. *(Yes it is, so if they can find the circumference, they can use this to find the area of the rectangle.)*

4. Ask the student to find the area of their circle. *(The formula for the area of the circle is πr^2 so they will find the radius of their circle and then substitute in the formula.)* Write the first part of this formula on a board:

   $SA_{Circle} = 2\ \pi r^2\ r^2$

   Students will multiply by two because they have a circle on the top and on the bottom.

5. Students will now find the area of the rectangle: $A = l \cdot w$. What if the label is on the can? Ask if there is a substitute that they can make for the length *(the circumference is the same as the length)*. Ask the students to remember how they found the circumference *($C = \pi r^2$)*.

6. Therefore, this part of the formula will be $SA_{Rectangle} = \pi d \cdot h$.

7. To find the surface area of the whole can, the students need to add the two calculations together to make the following formula:

   $SA = 2\ \pi r^2\ + (\pi d \cdot h)$

8. Show the above formula to the class.

9. Show the *Surface Area of a Cylinder* PowerPoint presentation.

10. For practice, give the students the *Surface Area of a Cylinder* student handout.

Answers for handout, Surface Area of Cylinders

1. 4 cm
2. 50.24 cm^2
3. 2
4. 301.44 cm^2
5. 401.92 cm^2
6. 10 cm
7. 78.5 cm^2
8. 314 cm^2
9. 471 cm^2
10. 307.72 ft^2
11. 879.2 ft^2
12. 1186.92 ft^2

H	A	D		A		B	Y	T	E	!	
471	10	1186.92	50.24	4	78.5	307.72	301.44	314	401.92	2	879.2

Evaluation

It is important for students to show their work on these exercises. There is obviously not room on the sheet itself, but they could use the back of the paper or a separate paper.

A percentage of the number correct could be used.

An authentic assessment might include the students actually writing or explaining aloud what they have done to solve these problems.

Key Points

Volume of a Rectangular Prism

- A rectangular prism has six sides
- The volume is the amount inside of the prism
- Volume is always a cubic measure
- The formula is $V = lwh$

Surface Area of a Rectangular Prism

- A rectangular prism has six sides
- Opposite sides are equal
- Surface area is the amount of material it would take to cover the shape
- The formula for area is $A = lw$
- A sample formula for surface area is $SA = 2lw + 2lw + 2lw$

Volume of a Cylinder

- Volume is the amount that can be held in the cylinder
- A cylinder is like a can. It has two equal circles on the top and the bottom.
- Radius is the distance from the center of the circle to the edge
- The formula for the area of a circle is $A = \pi r^2$
- The formula for the volume of a cylinder is $V = \pi r^2 \cdot h$

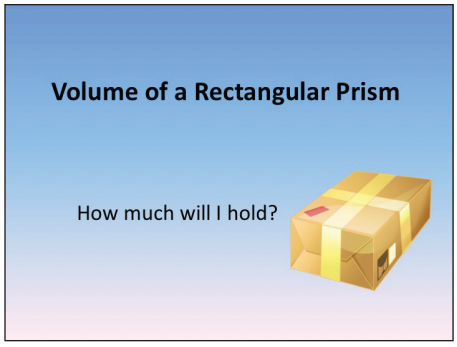
Surface Area of a Cylinder

- Surface area is the amount of material it would take to cover the cylinder
- Both the top and the bottom of the cylinder must be covered
- The formula for the area of a circle is $A = \pi r^2$ multiplied by 2 is $A = 2\pi r^2$
- The circumference of the circle is the same as the length of the can
- Diameter is the distance from one edge of the circle to the other through the midpoint
- The formula for the circumference of a circle is $C = \pi d$
- If you substitute this in $A = l \cdot w$, that would be $A = \pi d \cdot h$
- Therefore, this is the formula for the surface area of a circle:

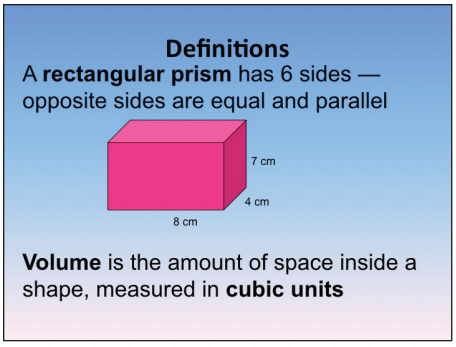
  2 circles + cover = SA

  $2\pi r^2 + \pi d \cdot h = SA$

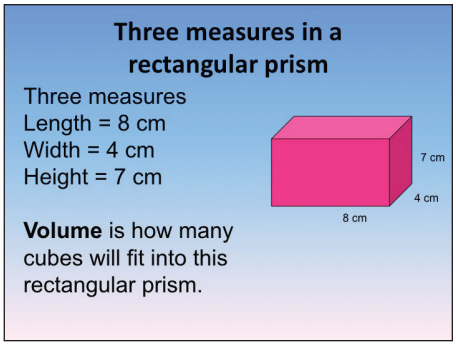
Volume of a Rectangular Prisms PowerPoint Presentation



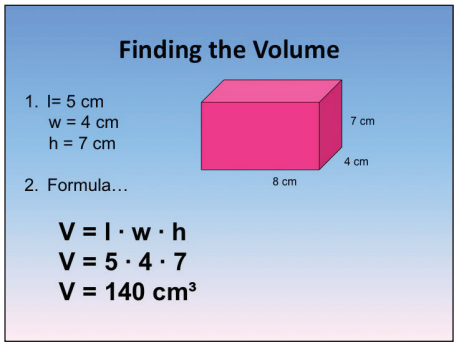
Slide 1



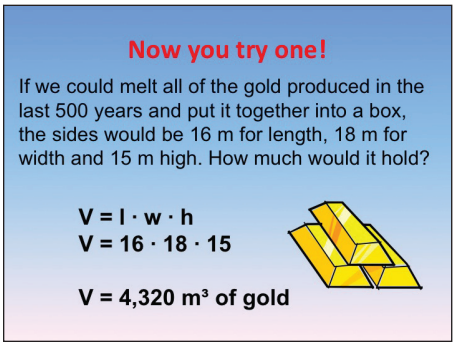
Slide 2



Slide 3



Slide 4



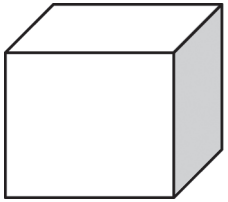
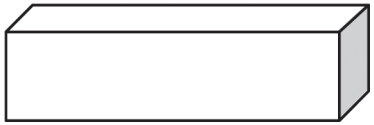
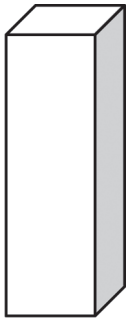
Slide 5

Name _____

Volume of a Rectangular Prism

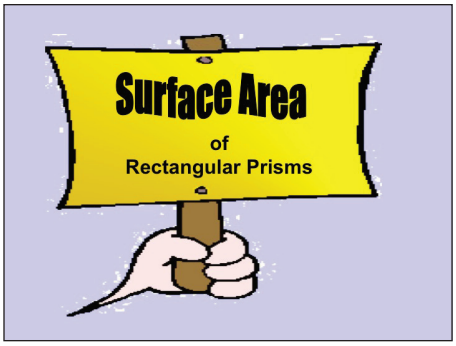
Find the volume of each prism and write the letter in the code below that matches the answers.

What does the tuba call its father?

1.  V = _____m^3 (P)	2.  V = _____ft^3 (M)	3.  V = _____m^3 (A)
4. A box of Oats Floats cereal is 24 cm wide, 30 cm high and 5 cm deep. What is the volume of the box? V = _____cm^3 (P)	5. A swimming pool is 30 ft long, 11 ft wide and 8 ft deep. What's the volume of the pool? V = _____ft^3 (O)	6. The Akron Airdock (for blimps) is 1000 ft long, 325 ft. wide and 30 ft high. What's the volume of the Airdock? V = _____ft^3 (A)
7. Which is correct... Volume = 5 ft Volume = 5 ft^2 Volume = 5 ft^3 (O) _____	8. True or False... Volume measures the area of every side of a rectangular prism. _____ (O)	9. How many sides does a rectangular prism have? _____ (O)

5 ft^2	5 ft^3	False	2640	600	True	216	270	3600	9,750,000	5000	7	5 ft	275

Surface Area of Rectangular Prisms PowerPoint Presentation



Slide 1

Surface Area

- What does it mean to you?
- Does it have anything to do with what is in the inside of the prism?
  VOLUME (not surface area) is the amount a shape can hold inside.
- **Surface area** is found by finding the area of all the sides and then adding those answers together.

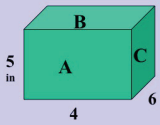
Slide 2

Surface Area of Rectangular Prisms

- What is area?
  The amount of square units that will COVER a shape.
- How will the answer be labeled?
  Units² **because it is area!**

Slide 3

Rectangular Prism



A = 5 x 4 = 20 x 2 =40
B = 6 x 5 = 30 x 2 = 60
C = 4 x 6 = 24 x 2 = 48

148 in²

How many faces are on here? 6

Find the area of each of the faces.

Do any of the faces have the same area?

If so, which ones?

Opposite faces are the same.

Find the SA

Slide 4

Cube

Are all the faces the same? YES

How many faces are there? 6

Find the surface area of one of the faces.

4 x 4 = 16
X 6
96 m²

Take that times the number of faces.

Slide 5

Now It's **YOUR** Turn!

I think I can!
I think I can!
I think I can!
I think I can!
I think I can!
I think I can!

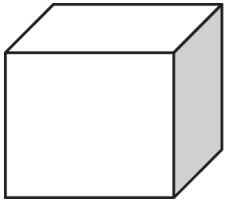
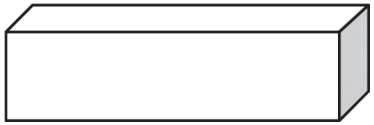
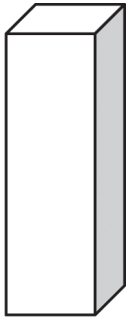
Slide 6

Name _____

Surface Area of a Rectangular Prism

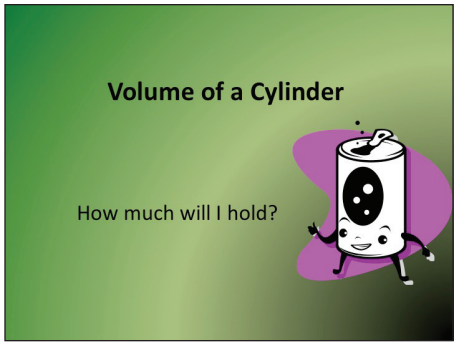
Find the surface area of each prism and write the letter in the code below that matches the answers.

What do elves learn at school?

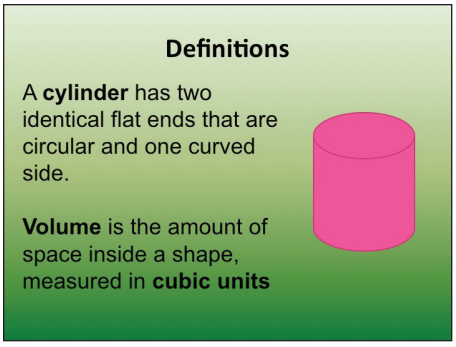
1.  SA = _____m^2 (E)	2.  SA = _____ft^2 (F)	3.  SA = _____m^2 (A)
4. A Valentine box is 12 inches long, 10 inches wide and 8 inches tall. How much paper is necessary to cover the box? SA = _____cm^2 (E)	5. A teacher made a box to hold the calculators. It was 20 inches long, 15 inches wide and 8 inches high. How much paper does she need to cover the box? SA = _____ft^2 (L)	6. A die hanging in your car is six inches on every side. What is the surface area of the die? SA = _____ft^2 (H)
7. Which is correct? SA = 5 ft SA = 5 ft^2 SA = 5 ft^3 (T) _____	8. True or false: Surface area measures the area of every side of a rectangular prism. _____ (E)	9. How many sides does a rectangular prism have? _____ (T) 10. Free Space (B)

5 ft^3	5 ft^2	216	592	False	236	1160	188	5 ft	164	Free Space	True	6

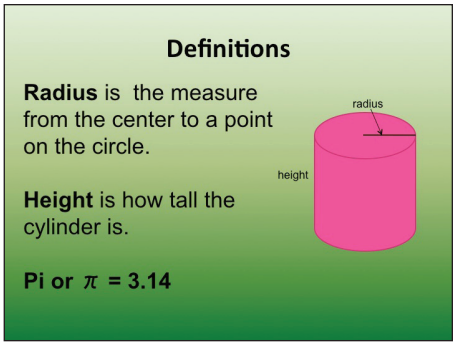
Volume of a Cylinder PowerPoint Presentation



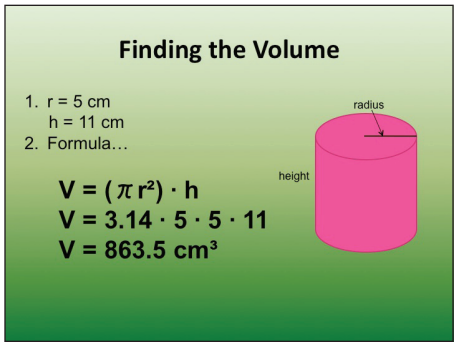
Slide 1



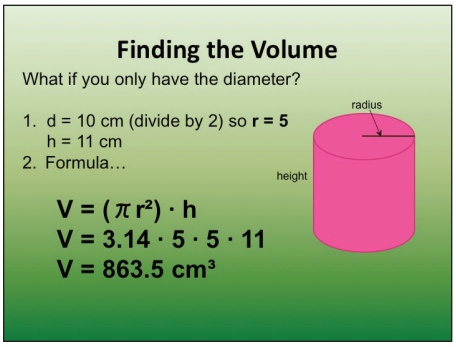
Slide 2



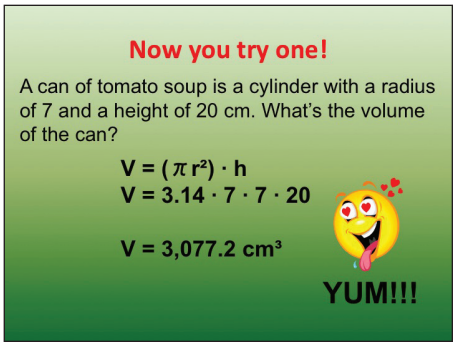
Slide 3



Slide 4



Slide 5



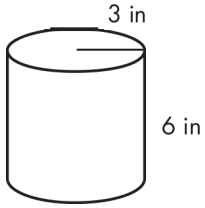
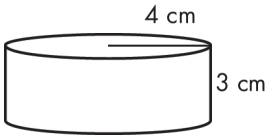
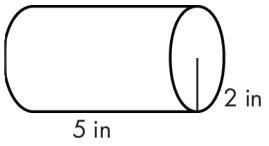
Slide 6

Name _____

Volume of a Cylinder

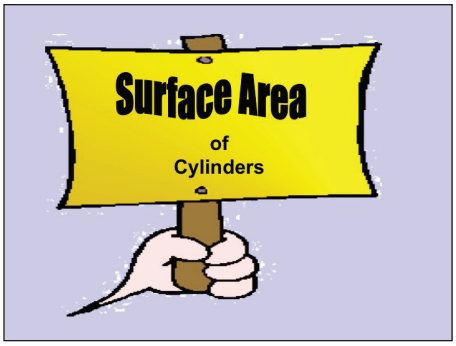
Find the volume of these cylinders. Find the answer in the answer column and write the letter above it. **Note:** Sometimes the area of the circles is given and sometimes only the radius is given.

What is a snake's favorite subject?

1.  V = _____ (T)	2.  V = _____ (H)	3.  V = _____ (S)
4. A soup can is 12 centimeters high. The radius of the top and bottom is 5 centimeters. What is the volume of the can? V = _____ (O)	5. One cubic foot holds 7.5 gallons of water. A cylindrical tank is five feet high and has a radius of two feet. About how many gallons of water does the tank hold? V = _____ (Y)	6. Calculate the volume of a cylinder with a radius of five centimeters and a height of 11 centimeters. V = _____ (S)
7. Which is correct? Volume = 10 ft Volume = 10 ft^2 Volume = 10 ft^3 _____ (–)	8. True or False... If you are given the diameter of the top of a can, you can still find the volume. _____ (True)	9. What is the volume of cylinder; radius 9 cm and height 15 cm? V = _____ (R)

10 ft	150.72	True	62.8	863.5	10 ft^3	169.56	942	3815.1	8.4	10 ft^2	False

Surface Area of a Cylinder PowerPoint Presentation



Slide 1

Surface Area

- What does it mean to you?
- Does it have anything to do with what is in the inside of the prism?
  VOLUME (not surface area) is the amount a shape can hold inside.
- **Surface area** is found by finding the area of the circle and the area around the cylinder and adding it together.

Slide 2

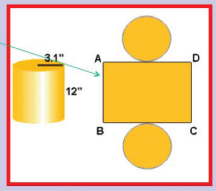
Surface Area of Cylinders

- What is area?
  The amount of square units that will COVER a shape.
- How will the answer be labeled?
  Units² **because it is area!**

Slide 3

SURFACE AREA of a CYLINDER.

Imagine that you can open up a cylinder like so



You can see that the surface is made up of two circles and a rectangle.

The length of the rectangle is the same as the circumference of the circle!

Slide 4

EXAMPLE: Round to the nearest TENTH.

Top or bottom circle

$A = \pi r^2$

$A = \pi (3.1)^2$

$A = \pi (9.61)$

$A = 30.2\ cm^2$

Rectangle

C = length
The length is the same as the Circumference

$C = \pi\ d$

$C = \pi (6.2)$

$C = 19.5\ cm$

Now add:

30.2 + 30.2 + 234 =

$SA = 294.4\ in^2$

Now the area

$A = lw$

$A = 19.5(12)$

$A = 234\ cm^2$

Slide 5

This could be written a different way.

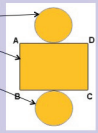
$SA = 2\pi rh + 2\pi r^2$

$A = \pi r^2$ (one circle) This is the area of the top and the bottom circles.

$2\pi r = \pi d$

So this formula could be written:

$SA = 2\pi r^2 + \pi d \cdot h$



Slide 6

There is also a formula to find surface area of a cylinder.

Some people find this way easier:

$SA = 2\pi rh + 2\pi r^2$

$SA = 2\pi rh + 2\pi r^2$

$SA = 2\pi (3.1)(12) + 2\pi (3.1)^2$

$SA = 2\pi\ (37.2) + 2\pi (9.61)$

$SA = \pi (74.4) + \pi (19.2)$

$SA = 233.7 + 60.4$

$SA = 294.1\ in^2$

The answers are REALLY close, but not exactly the same. That's because we rounded in the problem.

Slide 7

Now It's **YOUR** Turn!

I think I can!
I think I can!
I think I can!
I think I can!
I think I can!
I think I can!

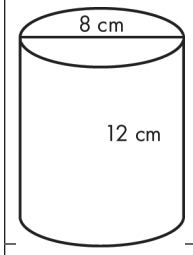
Slide 8

Name _____

Surface Area of Cylinders

Do each exercise and find your answer in the answer column. Write each letter above the answer below to answer the question,

What did the computer do at lunchtime?

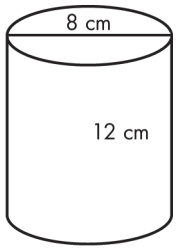


1. What is the radius of the circle for this can?

   r = _____ (A)

2. What is the area of the circle?

   A_{circle} = _____ (-)

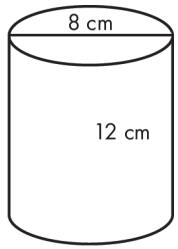


3. How many circles do you need for the SA?

   _____ (!)

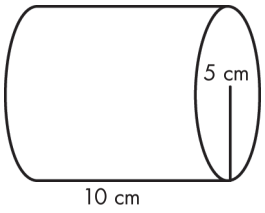
4. What is the area of the cover around for this can? A =

   _____ (Y)



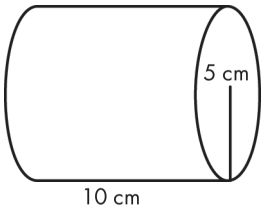
5. What is the surface area of this cylinder?

   Surface Area =_____ (E)



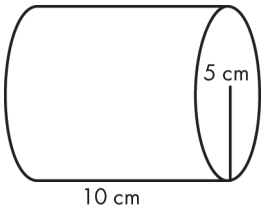
6. What is the diameter of the circle for this can?

   d = _____ (A)



7. What is the area of the circle for this can?

   $Area_{circle}$ = _____ (-)



8. What is the area of the cover?

   A = _____ (T)

9. What is the surface area of this cylinder?

   SA = _____ (H)

The top, bottom and sides of a cylindrical tank with a diameter of 14 feet and a height of 20 feet are to be painted. How much surface area is to be painted?

10. What is the area of the top and bottom (two circles)?

    A = _____ (B)

11. What is the area of the side (cover)?

    A = _____

    (-)

12. 12. What is the surface area that is to be painted?

    SA = _____

    (D)

471	10	1186.92	50.24	4	78.5	307.72	301.44	314	401.92	2	879.2

Introduction to Surface Area and Volume

Popcorn Prisms

Overview

Using two sheets of paper, students will create two rectangular prisms, one using the 11" side as the height and the second using the 8.5" side as the height. They will then predict which will hold more popcorn and calculate to determine if their prediction was correct.

Standards Addressed

Measurement

Grade 6

05-07 Benchmark

G. Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes.

Y2003.CMA.S02.G05-07.BG.L06.I05
Use Measurement Techniques and Tools /

05. Understand the difference between perimeter and area, and demonstrate that two shapes may have the same perimeter, but different areas or may have the same area, but different perimeters.

05-07 Benchmark

F. Analyze and explain what happens to area and perimeter or surface area and volume when the dimensions of an object are changed.

Y2003.CMA.S02.G05-07.BF.L06.I06
Use Measurement Techniques and Tools /

06. Describe what happens to the perimeter and area of a two-dimensional shape when the measurements of the shape are changed; e.g. length of sides are doubled.

Materials

- Formative assessment *What Am I? Vocabulary* (either as handout or a PowerPoint presentation)
- *Popcorn Prisms* PowerPoint presentation and handout
- Paper
- Large quantity of popcorn
- Tape
- Calculators

Procedure

1. Perform a formative assessment on vocabulary by using the *What Am I?* student handout or PowerPoint presentation. Review answers with the students. *Answers: 1. C, area; 2. B, circumference; 3. volume; 4. surface area; 5. diameter; 6. hypotenuse; 7. radius; 8. polygon; 9. perimeter; 10. C, pi or π; 11. volume.*

2. Show the video *Surface Area and Volume* and discuss the difference between surface area and volume.

3. Break the students into groups of two or three. Give each group two sheets of 8.5" x 11" inch paper, preferably of different colors.

4. Review finding surface area and volume with the students. Tell them that in this problem, they would only be finding the surface area of the bottom of the prism (not the top), so they're finding the surface area of five sides.

5. Review the *Popcorn Prism* PowerPoint presentation and distribute the *Popcorn Prism* handout.

6. Tell the students that they must make a prediction before they begin.

7. They will then fill in the table and answer questions 4 and 5, but wait to complete the lesson until the tables have been discussed.

8. Go over the handout with the students. Start by asking how many predicted correctly. Ask them why they thought this even before they got empirical data that proved their prediction.

9. Ask students to explain how they got the answers and have them correct their work.

Answers

Measurements					
Figure Name	**Bottom Length**	**Bottom Width**	**Area of Bottom**	**Height of Side**	**Area of Sides**
Figure 1	2 1/8 or 2.125	2 1/8 or 2.125	2.125 x 2.125 = 4.5 sq in	11	4 · 2.125 · 11 93.5 sq in
Figure 2	2 3/4 or 2.75 in	2 3/4 or 2.75 in	2.75 x 2.75 = 7.56 sq in	8.5	4 · 2.75 · 8.5 = 93.5 sq in

Please show your work when you do these calculations.

Calculate the Surface Area Surface Area of This Rectangular Prism = 4 · (l x w) + l x w (of bottom) Volume = l · w · h		
	Surface Area = Area of Sides + Area of Bottom	**Volume = l · w · h**
Figure 1	93.5 + 4.5 = 98 sq in	2.125 • 2.125 • 11 **= 49.7 cu in**
Figure 2	93.5 + 7.56 = 101.1 sq in	2.75 • 2.75 · 8.5 **= 64.3 cu in**

4. Figure 2 had a greater surface area.

5. Figure 2 had a greater volume.

6. Figure 2 is not full. There is space for more popcorn.

Evaluation

This is an introduction to the concept of surface area and volume, so this could be used as an investigation.

The student handout can be quantified by giving six points for each of the 10 answers in the first table and 10 points for each answer in the second table, giving a total of 100 points.

Adapted from http://illuminations.nctm.org/Lessons/Popcorn/Popcorn-AK-Cylinders.pdf

Popcorn Prisms PowerPoint Presentation

Popcorn Prisms

Surface Area & Volume

Slide 1

To do the next two lessons, you need to know...

- That a **prism** is a 3-dimensional shape with 2 identical parallel bases.
- The formulas for SA (surface area) and for V (volume) of a rectangular prism and a cylinder.

Slide 2

Let's start in the beginning…

Before you can do surface area or volume, you have to know the following formulas.

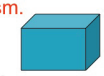
Rectangle $A = lw$

Circle $A = \pi r^2$

$C = \pi d$

Slide 3

Let's start with a rectangular prism.



Surface area can be done using the formula

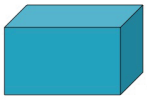
SA = 2 lw + 2 lw + 2 lw *OR* you can find the area for each surface and add them up.

Either method will give you the same answer.

Volume of a rectangular prism is V = lwh

Slide 4

Example:



Surface Area

Front/back 2(8)(4) = 64 cm²

Left/right 2(4)(7) = 56 cm²

Top/bottom 2(8)(7) = 112 cm²

Add them up!

SA = 232 cm²

Volume

V = lwh

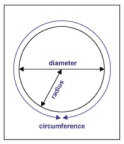
V = 8(4)(7)

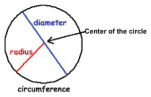
V = 224 cm³

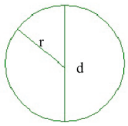
Slide 5

CIRCLES

You must know the difference between RADIUS and DIAMETER.



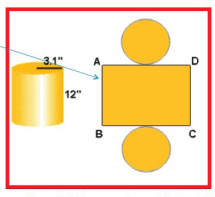




Slide 6

SURFACE AREA of a CYLINDER.

Imagine that you can open up a cylinder like so



You can see that the surface is made up of two circles and a rectangle.

The length of the rectangle is the same as the circumference of the circle!

Slide 7

EXAMPLE: **Round to the nearest TENTH.**

Top or bottom circle

$A = \pi r^2$

$A = \pi (3.1)^2$

$A = \pi (9.61)$

$A = 30.2\ cm^2$

Rectangle

C = length

$C = \pi\ d$

$C = \pi (6.2)$

$C = 19.5\ cm$

Now the area

A = lw

A = 19.5(12)

$A = 234\ cm^2$

Now add:

30.2 + 30.2 + 234 =

$SA = 294.4\ in^2$

Slide 8

There is also a formula to find surface area of a cylinder.

$SA = 2\pi rh + 2\pi r^2$

Some people find this way easier:

$SA = 2\pi rh + 2\pi r^2$

$SA = 2\pi (3.1)(12) + 2\pi (3.1)^2$

$SA = 2\pi\ (37.2) + 2\pi (9.61)$

$SA = \pi (74.4) + \pi (19.2)$

$SA = 233.7 + 60.4$

$SA = 294.1\ in^2$

The answers are REALLY close, but not exactly the same. That's because we rounded in the problem.

Slide 9

Find the radius and height of the cylinder.

The formula tells you what to do!!!!

Remember the order of operations. You multiply before you add.

$2\pi rh + 2\pi r^2$ means multiply $2(\pi)(r)(h) + 2(\pi)(r)(r)$

Slide 10

Volume of a Cylinder

We used this drawing for our surface area example. Now we will find the volume.

$V = (\pi r^2)(h)$

$V = (\pi)(3.1^2)(12)$

optional step! → $V = (\pi)(3.1)(3.1)(12)$

$V = 396.3 \text{ in}^3$

Slide 11

ARE YOU READY?

Let's see what you can do with POPCORN PRISMS!

Slide 12

Name _____

Popcorn Prisms

Your Task: You are to build two prisms using the paper you were given. See below.

Figure 1

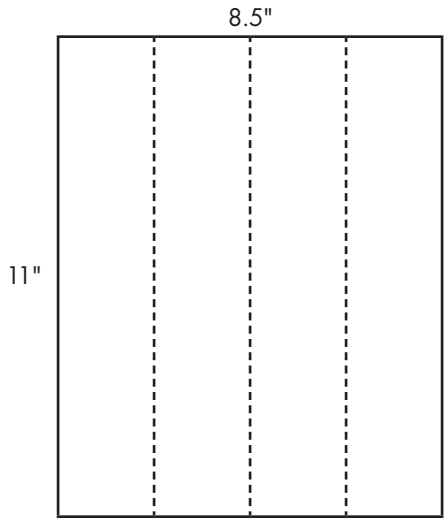
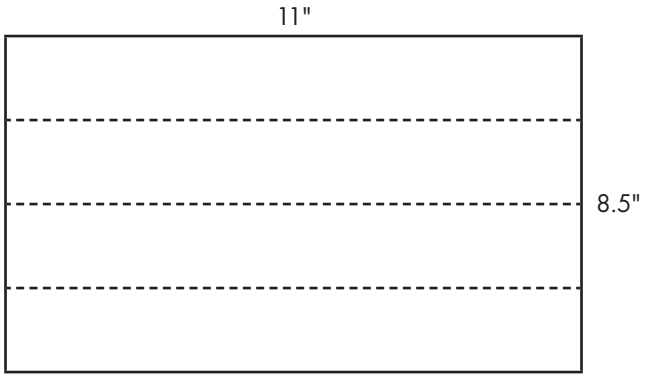


Figure 2



Directions:

a. Fold one sheet in half and then in half again so that you have four equal sections (as in Figure 1).

b. Open this up and fold it into a box and tape it. Fold the second sheet in half and then in half again as in Figure 2. Once again, fold into a rectangular prism and tape it.

c. Make a bottom for your prism and tape it on.

1. Each member of the team now needs to make a prediction as to which container (Figure 1 or Figure 2) will hold more popcorn.

Name	**Prediction**
	Figure 1 or Figure 2 or Same
	Figure 1 or Figure 2 or Same
	Figure 1 or Figure 2 or Same

2. Now let's find out the surface area and volume of each prism.

Measurements					
Figure Name	**Bottom Length**	**Bottom Width**	**Area of Bottom**	**Height of Side**	**Area of Sides**
Figure 1					
Figure 2					

Please show your work when you do these calculations.

Calculate the Surface Area Surface Area of This Rectangular Prism = 4 · (l x w) + l x w (of bottom) Volume = l · w · h		
	Surface Area = Area of Sides + Area of Bottom	**Volume = l · w · h**
Figure 1		
Figure 2		

3. Did one have a greater surface area than the other? If so, which one?

4. Did one have a greater volume than the other? If so, which one?

5. You have shown mathematically that one has a greater volume than the other. Now let's prove it.
    a. Fill Figure 1 with popcorn and place it inside Figure 2. Make sure that Figure 1 is full.

    b. Now empty Figure 1 into Figure 2.

    c. What do you observe?

6. Were you correct in your prediction? Are you surprised by what happened?

Introduction to Surface Area and Volume

Popcorn Cylinders

Overview

Using two sheets of paper, student will create two cylinders, one using the 11" side as the height and the second using the 8.5" side as the height. They will then predict which will hold more popcorn and calculate to determine if their prediction was correct.

Standards Addressed

Measurement

Grade 6

05-07 Benchmark

G. Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes.

Y2003.CMA.S02.G05-07.BG.L06.I05 Use Measurement Techniques and Tools /

05. Understand the difference between perimeter and area, and demonstrate that two shapes may have the same perimeter, but different areas or may have the same area, but different perimeters.

Materials

- Two sheets of paper per person
- Large quantity of popcorn
- Tape
- Calculators
- Rulers

Procedure

1. Review with the students what they learned from making rectangular prisms.
2. Review finding the area and circumference of a circle. Show the PowerPoint presentation *Popcorn Prisms* again, if necessary.
3. Break the students into groups of two or three. Give each group two sheets 8.5" x 11" paper, preferably of different colors.

4. Review finding surface area and volume of a cylinder with the students. Tell them that it will be difficult to create the base for this cylinder, so they can simply use a ruler and estimate the diameter of that circle and use that to calculate the radius.

5. Distribute the *Popcorn Cylinders* student handout.

6. Tell the students that they must make a prediction before they begin.

7. Instructs the students to fill in the table and answer questions 4 and 5, but wait to complete the lesson until the tables have been discussed.

Answers

Calculate the volume of each cylinder. Volume = $\pi r^2 \cdot h$				
	Diameter	**Radius**	**Substitute below** **V =** $\pi \cdot r \cdot r \cdot h$	**Volume**
Figure 1	@ 2.7	@ 1.4	3.14 · 1.4 · 1.4 · 11 =	@ 67.7 in^3
Figure 2	@ 3.4	@ 1.7	3.14 · 1.7 · 1.7 · 8.5 =	@ 77.1 in^3

4. Figure 2 had a greater volume.

5. The measurement of the diameter and radius varied from group to group. Therefore, the volumes would be slightly different.

6. Figure 2 is not full. There is space for more popcorn.

Back to the Lesson

8. Go over the handout with the students. Start by asking how many predicted correctly. Ask them if they made a better prediction than they did when they worked with rectangular prisms.

9. Ask students to explain how they got the answers and have them correct their work.

Evaluation

The popcorn problems could be used as authentic assessment. If students complete the task, they can be asked to write their understanding of the process by which both activities were done. They can then be graded using the following rubric.

CATEGORY	4	3	2	1
Strategy/Procedures	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Reasoning	Uses complex and refined mathematical reasoning.	Uses effective mathematical reasoning.	Some evidence of mathematical reasoning.	Little evidence of mathematical reasoning.
Use of Manipulatives	Student always listens and follows directions and only uses manipulatives as instructed.	Student typically listens and follows directions and uses manipulatives as instructed most of the time.	Student sometimes listens and follows directions and uses manipulatives appropriately when reminded.	Student rarely listens and often "plays" with the manipulatives instead of using them as instructed.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.

Adapted from http://illuminations.nctm.org/Lessons/Popcorn/Popcorn-AK-Cylinders.pdf

Name _____

Popcorn Cylinders

Your Task: You are to build two prisms using the paper you were given. See below.

Figure 1

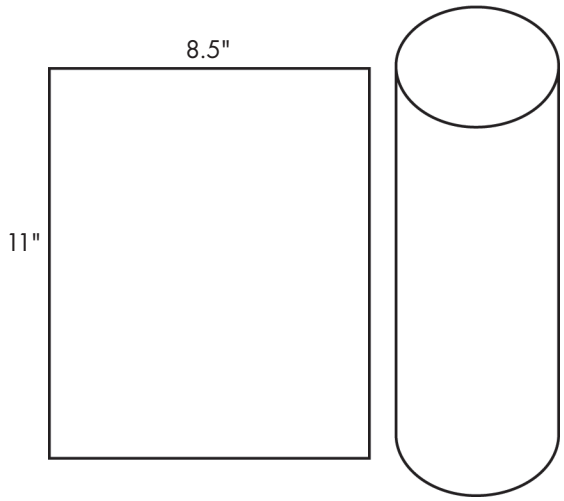
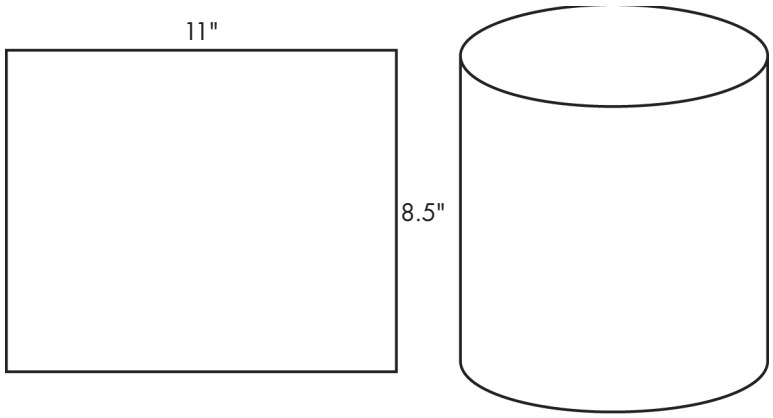


Figure 2

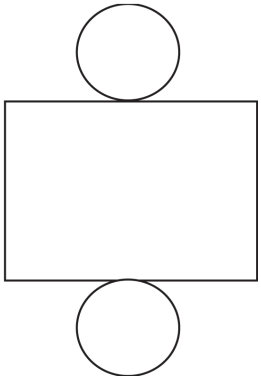


1. Directions:
    a. Make a cylinder with each paper (as in Figure 1) and tape it together on the side..
    b. Make a bottom for your prism and tape it on.

2. Think about what you learned when you made rectangular prisms. Each member of the team now needs to make a prediction as to which container (Figure 1 or Figure 2) will hold more popcorn.

Name	**Prediction**
	Figure 1 or Figure 2 or Same
	Figure 1 or Figure 2 or Same
	Figure 1 or Figure 2 or Same

If the cylinder is flattened out, it will look like this.



REMEMBER:

The length of the rectangle will be the same as the circumference of the circle.

3. We don't need the surface area to determine which cylinder will hold more popcorn. We need the volume.

4. It is difficult to create an actual base for this figure, so tape the papers together at the top and the bottom and estimate the diameter. Use that estimate to find the radius.

Please show your work when you do these calculations.

Calculate the volume of each cylinder. Volume = $\pi r^2 \cdot h$				
	Diameter	**Radius**	**Substitute below** **V =** $\pi \cdot r \cdot r \cdot h$	**Volume**
Figure 1				
Figure 2				

5. Did one have a greater volume than the other? If so, which one?

6. Why did each group come up with slightly different answers?

7. You have shown mathematically that one has a greater volume than the other. Now let's prove it.
    a. Fill Figure 1 with popcorn and place it inside Figure 2. Make sure that Figure 1 is full.
    b. Now empty Figure 1 into Figure 2.
    c. What do you observe?

8. Were you correct in your prediction? Was your prediction better today than when you worked with rectangular prisms?

Summative Assessment

Volume and Surface Area

1. What is volume?

2. What is surface area?

3. Write two facts about rectangular prisms.

   a.

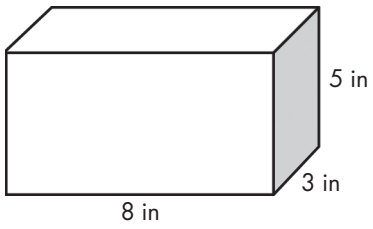
   b. .

4. Write two facts about cylinders.

   a.

   b. .

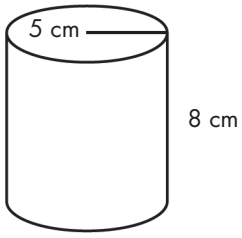
Given this rectangular prism:



5. Find the volume. Show all work.

6. Find the surface area. Show all work.

Given this cylinder:



7. What is the radius? _____

8. What is the area of the circle? Show all work.

9. What is the volume of the cylinder? Show all work.

10. What is the diameter of the circle? _____

11. What's the circumference of the circle? _____

12. What is the surface area of the cylinder? Show all work.

13. The measure of a volume is always **squared** or **cubed** (select one).

14. The measure of surface area is always **squared** or **cubed** (select one).

BONUS:

Write the formula for the volume of a rectangular prism.

Write the formula for the surface area of a rectangular prism.

Write the formula for the volume of a cylinder.

Write the formula for the surface area of a cylinder.

Summative Assessment — Answers

Volume and Surface Area

1. What is volume?

   Amount inside a shape

2. What is surface area?

   Amount it takes to cover a shape

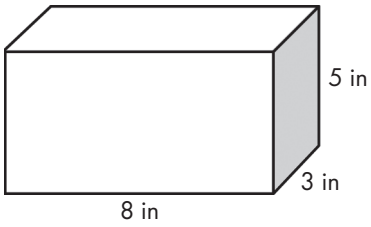
3. Write two facts about rectangular prisms.

   a. *has six sides*
   b. *opposite sides are equal*
   c. *angles are 90 degrees*
   d. *opposite sides are parallel*
   e. *has length, width and height*

4. Write two facts about cylinders.

   a. *looks like a can*
   b. *has two circles (top and bottom)*
   c. *circles are equal size*
   d. *has a cover or a height*

Given this rectangular prism:



5. Find the volume. Show all work.

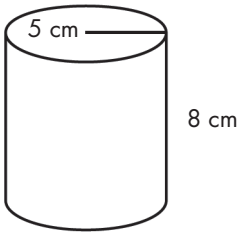
   $V = l \cdot w \cdot h$
   8 x 3 x 5 = 120 in³ or 120 cubic inches

6. Find the surface area. Show all work.

   $SA = 2lw + 2lw + 2lw$
   (2 x 8 x 3) + (2 x 3 x 5) + (2 x 8 x 5)
   48 + 30 + 80

   SA = 150 in² or 150 square inches

Given this cylinder:



7. What is the radius?

   5 cm

8. What is the area of the circle? Show all work.

   $A = \pi r^2$
   A = 3.14 x 5 x 5
   A = 78.5 cm² or 78.5 square cm

9. What is the volume of the cylinder? Show all work.

   $V = \pi r^2 \cdot h$
   (3.14 x 5 x 5) x 8
   V = 628 cm³ or 628 cubic cm

10. What is the diameter of the circle?

    10 cm

11. What's the circumference of the circle?

    $C = \pi d$
    C = 3.14 x 10
    C = 31.4 cm²

12. What is the surface area of the cylinder? Show all work.

    $SA = (2 \cdot \pi r^2) + (\pi \cdot d \cdot h)$
    (2 · 3.14 · 5 · 5) + (3.14 · 10 · 8)
    157 + 251.2
    408.2 cm²

13. The measure of a volume is always **squared** or **cubed** (select one).

14. The measure of surface area is always **squared** or **cubed** (select one).

BONUS:

(There are variations of these formulas. As long as they express the same result, they are acceptable.)

Write the formula for the volume of a rectangular prism.

$V = l \cdot w \cdot h$

Write the formula for the surface area of a rectangular prism.

$SA = 2lw + 2lw + 2lw$

Write the formula for the volume of a cylinder.

$V = \pi r^2 \cdot h$

Write the formula for the surface area of a cylinder.

$SA = (2 \cdot \pi \cdot r^2) + (\pi \cdot d \cdot h)$

Evaluation

1. A percent of correct answers could be used to give a grade.

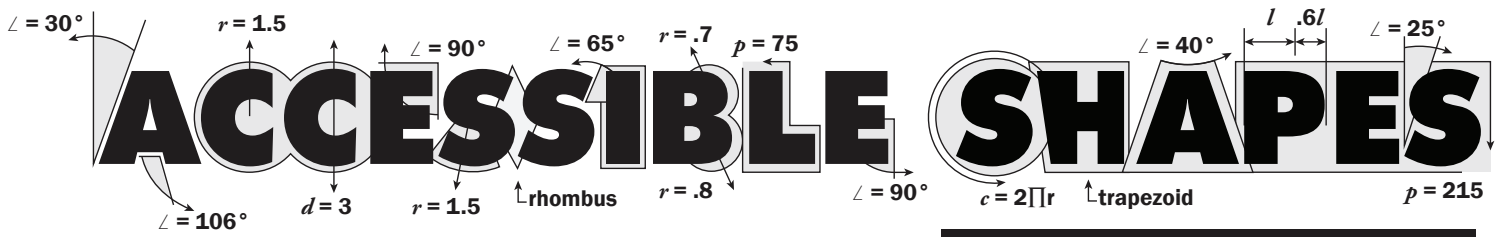
2. Students could work in teams to come up with acceptable answers.

3. It could be worked through as a study sheet over everything that has been covered and then one or two problems could be given to determine understanding.

Alternative Assessment

The popcorn problems could be used as authentic assessment. If students complete the task, they can be asked to write their understanding of the process by which both activities were done. They can then be graded on the following rubric.

CATEGORY	4	3	2	1
Strategy/Procedures	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Reasoning	Uses complex and refined mathematical reasoning.	Uses effective mathematical reasoning	Some evidence of mathematical reasoning.	Little evidence of mathematical reasoning.
Use of Manipulatives	Student always listens and follows directions and only uses manipulatives as instructed.	Student typically listens and follows directions and uses manipulatives as instructed most of the time.	Student sometimes listens and follows directions and uses manipulatives appropriately when reminded.	Student rarely listens and often "plays" with the manipulatives instead of using them as instructed.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.


∠ = 30°
r = 1.5
∠ = 90°
∠ = 65°
r = .7
p = 75
l
.6l
∠ = 40°
∠ = 25°
ACCESSIBLE SHAPES
∠ = 106°
d = 3
r = 1.5
rhombus
r = .8
∠ = 90°
c = 2∏r
trapezoid
p = 215
GEOMETRY

Hotlist

Surface Area and Volume Hotlist

General

AAA Math: Geometry
http://aaaknow.com/geo.htm#topic22

Areas, Volumes, Surface Area Gives formulas and drawings for each polygon
http://math2.org/math/geometry/areasvols.htm

A Math Dictionary for Kids
http://www.teachers.ash.org.au/jeather/maths/dictionary.html

Computing Technology for Math Excellence – VERY comprehensive site with material for all grade levels
http://www.ct4me.net/

IXL Interactive practice for all sixth grade math topics (other grades available by changing the ending of the URL)
http://www.ixl.com/math/grade/sixth/

Math for Morons Like Us: Geometry From ThinkQuest
http://library.thinkquest.org/20991/geo/index.html

Naming Polygons and Polyhedra
http://mathforum.org/dr.math/faq/faq.polygon.names.html

Perimeter, Area, Surface Area and Volume Resource sheet with definitions and pictures
http://www2.scholastic.com/content/collateral_resources/pdf/smp/actuarial/actuarial4_ref_sheet.pdf

Rhythm, Rhyme Results Rap song about surface area. Lyrics are also printed
http://www.educationalrap.com/song/its-all-in-the-surface.html

WebQuests

Using Area and Perimeter to Design a Fun House
http://its.guilford.k12.nc.us/webquests/areaperim/areaperim.htm

Manipulatives and Games

National Gallery of Virtual Manipulatives
http://nlvm.usu.edu/en/nav/category_g_3_t_3.html

Cyberchase: Star Gazing – Measures angles
http://pbskids.org/cyberchase/games/anglemeasurement/anglemeasurement.html

Videos

What's the Problem: Changing Spaces — Really cute video showing the problems college students have when they mix up surface area and volume. There are related online activities
http://www.woub.org/wtp/changing_spaces/index.htm

Videos — This site has many videos for volume and surface area. They're short and well done.
http://www.youtube.com/watch?v=cdsPFCMmjkw

Formulas

Formulas for Geometric Figures
http://www.science.co.il/formula.asp

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